Filter Design for Radar Tracking of Maneuvering Targets

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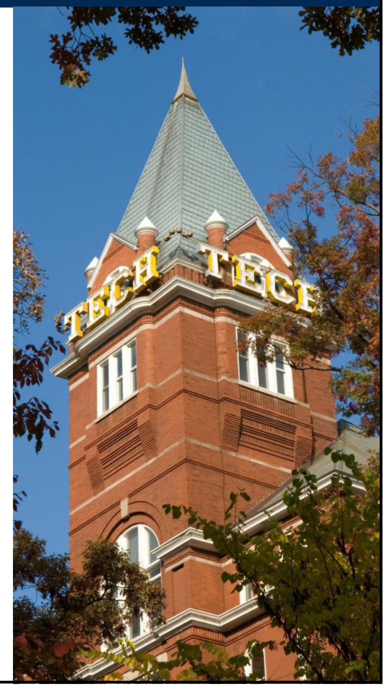

Outline

- **Introduction**
- Nearly-Constant Velocity (NCV) Track Filter Design
- Nearly Constant Acceleration (NCA) Track Filter Design
- Track Filter Design for Radar Tracking
- Filter Design and IMM Estimator
- Concluding Remarks

Introduction

- Estimation algorithms are most often designed and analyzed as unbiased estimators.
  - Cramer Rao Lower Bound (CRLB) is for unbiased estimators (biased version exists, but not commonly used)
  - For an unbiased estimator, the covariance generated by the estimator correctly characterizes the performance of the estimator.
- Kalman filter provides the minimum variance, unbiased estimate of the state of a system defined by

Constraint on the kinematic motion

State at time $k+1$ includes position, velocity, and possibly acceleration*

$$X_{k+1} = F_k X_k + G_k v_k$$

White Gaussian errors for system state process or maneuvers $v_k \sim N(0, Q_k)$

with measurements by

$$Z_k = H_k X_k + w_k$$

White Gaussian errors in the measurements $w_k \sim N(0, R_k)$

Introduction

- For tracking maneuvering targets, maneuvers are models as a white noise random process, while target maneuvers tend to be highly correlated or deterministic errors.
- Since the error covariance of the Kalman filter tends to be inconsistent for highly maneuvering targets, optimal filter design (i.e., selection of the process noise variance) is not immediate.
  - Targets are not maneuvering: Covariance is too large!
  - Targets are maneuvering: Covariance is too small!
- Process noise variance is often selected to be as small as possible while providing acceptable performance during maneuvers. How small is too small?
- Process noise variance can be selected sufficiently large to provide good performance during maneuvers. How large it too large?
- During maneuvers, the NCV Kalman filter is a biased estimator.
  - Treating the Kalman filter as an unbiased estimator with a covariance is invalid.
  - Mean Squared Error (MSE) is the correct performance metric, but only covariance is calculated.
  - Maximum MSE (MaxMSE) can be computed for target maneuvering with maximum acceleration (A_{max}).
  - Minimum MaxMSE (MinMaxMSE) can be used as a filter design criteria.
  - MaxMSE < Measurement error variance is a second design criteria.
- Nearly Constant Acceleration (NCA) filters can be designed similarly.

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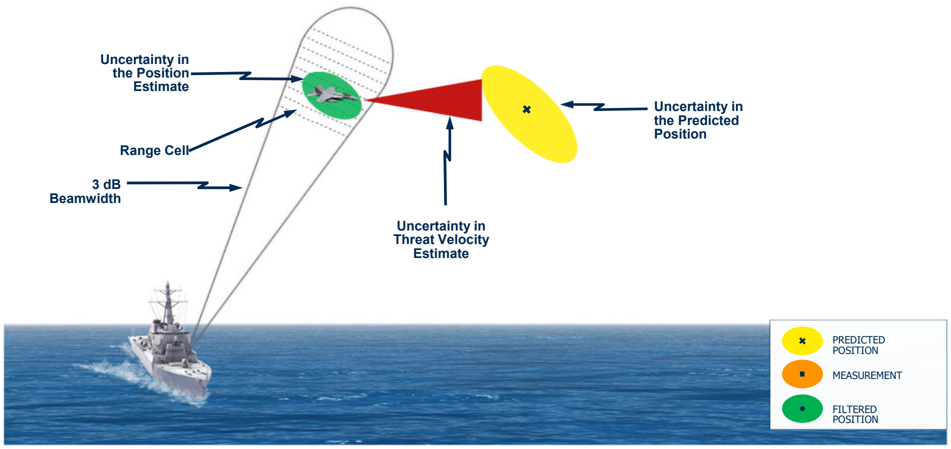
Target Tracking Process



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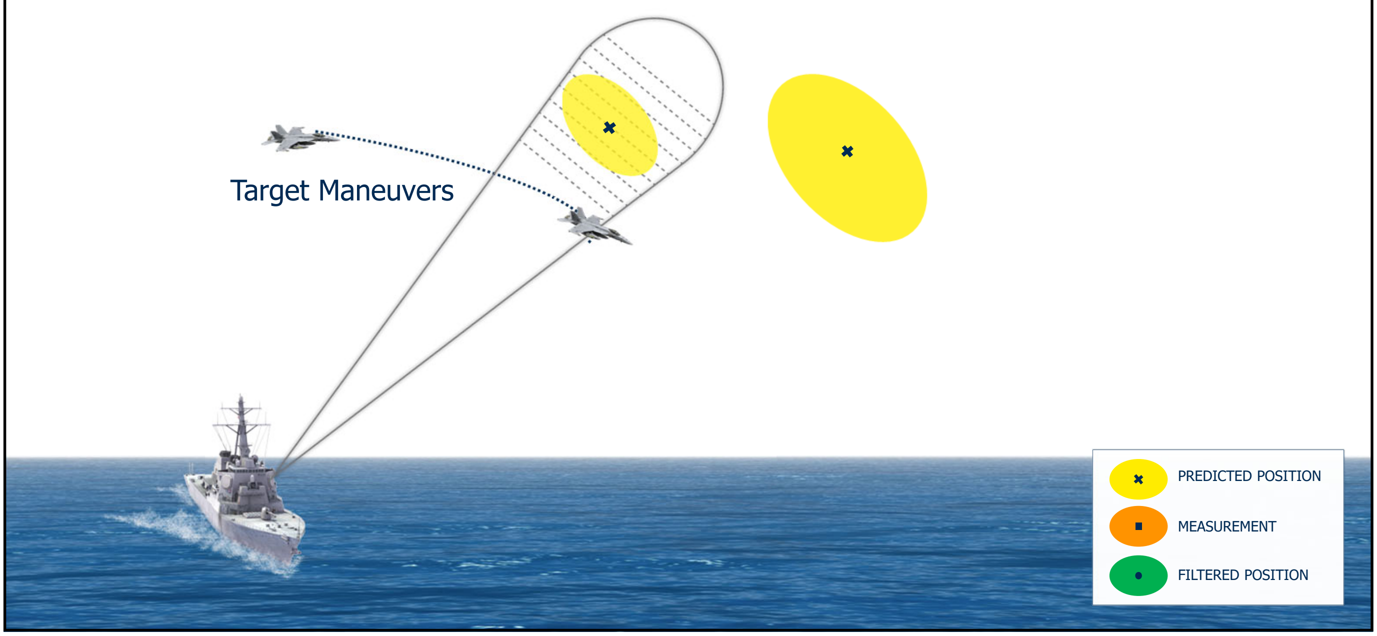
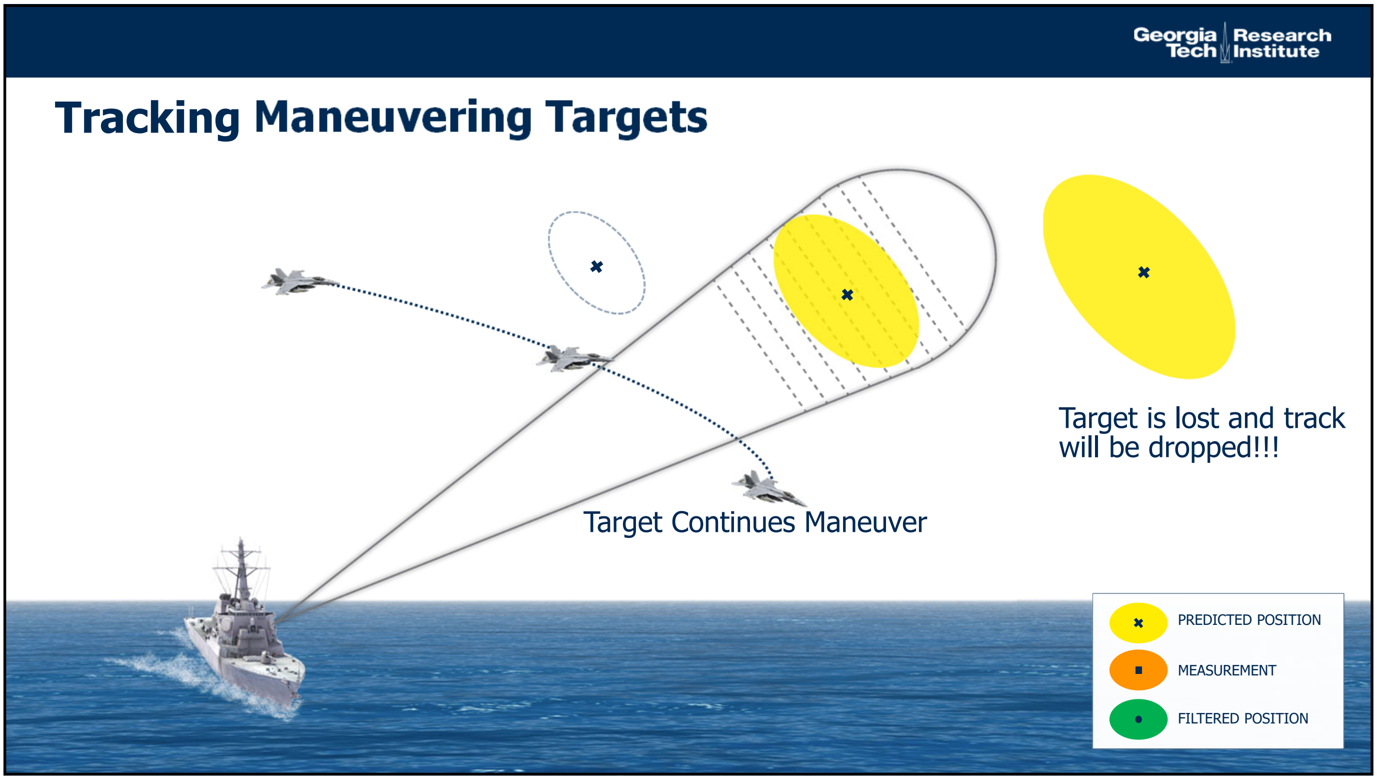
Tracking Maneuvering Targets

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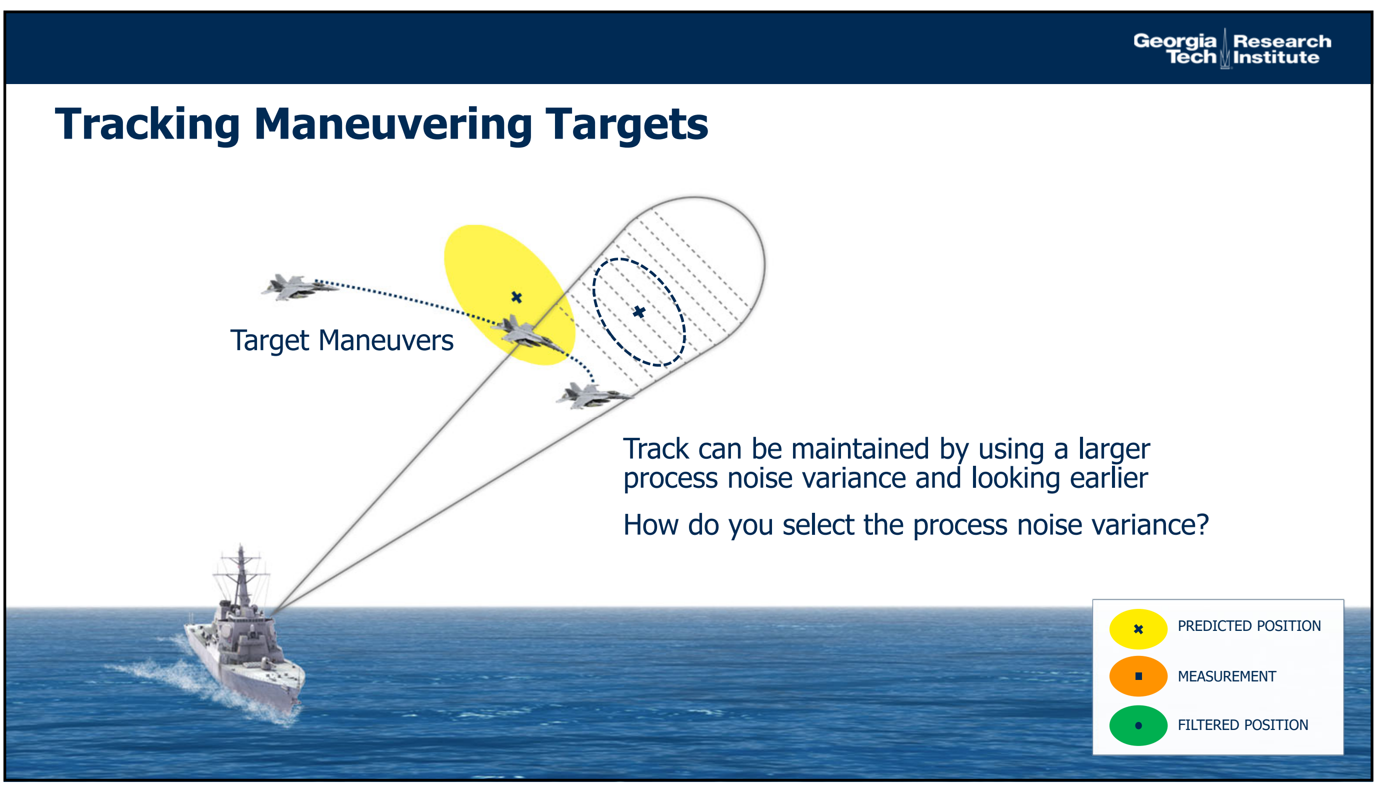
Tracking Maneuvering Targets
Target is lost and track will be dropped!!!
Target Continues Maneuver
PREDICTED POSITION
MEASUREMENT
FILTERED POSITION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Tracking Maneuvering Targets
Target Maneuvers
Track can be maintained by using a larger process noise variance and looking earlier
How do you select the process noise variance?
PREDICTED POSITION
MEASUREMENT
FILTERED POSITION

Target Motion Models

Typical forms of a state vector at time k for tracking a target in a scalar coordinate x

Stationary Target: $X_{k+1} = x_{k+1} = x_k = F_k X_k$

Position is modeled to remain fixed

Constant Velocity Target: $\dot{x}_{k+1} = \dot{x}_k$

Velocity is modeled to remain fixed

$$X_{k+1} = \begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & t_{k+1} - t_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} = F_k X_k$$

Constant Accleration Target: $\ddot{x}_{k+1} = \ddot{x}_k$

Acceleration is modeled to remain fixed

$$X_{k+1} = \begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \\ \ddot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & t_{k+1} - t_k & 0.5(t_{k+1} - t_k)^2 \\ 0 & 1 & t_{k+1} - t_k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} = F_k X_k$$

Higher order models add complexity and computational cost to the filter, with little or no benefit or even, sometimes with degraded performance.

Track Filter Basics

- Typical forms of a typical state vector at time k for tracking a target are

$$X_k = [x_k] \quad \text{stationary target}$$

$$X_k = [x_k \quad \dot{x}_k]^T \quad \text{constant velocity target}$$

$$X_k = [x_k \quad \dot{x}_k \quad \ddot{x}_k]^T \quad \text{constant acceleration target}$$

- The estimate of the state at time *k* given measurement to time *j* denoted as

$$X_{k|j} = [x_{k|j}] \qquad X_{k|j} = [x_{k|j} \quad \dot{x}_{k|j}]^T \qquad X_{k|j} = [x_{k|j} \quad \dot{x}_{k|j} \quad \ddot{x}_{k|j}]^T$$

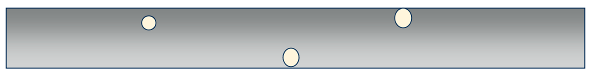
$X_{k|k}$ = denotes the filtered state estimate

$X_{k|k-1}$ = denotes the one-step predicted state estimate

$X_{k|k+1}$ = denotes the one-step smoothed state estimate

Track Filtering: Mathematics Versus Engineering

- Mathematics tells us that you can draw a straight line through any two points.
- In practice, we as engineers know that you can draw a straight line through any three points.



- **You just need a sufficiently wide pencil.**

Constant Velocity (CV) or Constant Acceleration (CA)?

First and only contribution to answering this question is presented in W. D. Blair and Y. Bar-Shalom, "On the NCA Versus NCV Models in Tracking of Maneuvering Targets," ***Proceedings of the 2023 IEEE Radar Conference,*** **San Antonio, Texas, May 2023**

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Nearly Constant Velocity (NCV) Motion Model with Discrete White Noise Acceleration (DWNA)

NCV filter is the most widely used for target tracking.

Motion model:

$$X_k = \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} = F_{k-1}X_{k-1} + G_{k-1}v_{k-1} = \begin{bmatrix} 1 & t_k - t_{k-1} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ \dot{x}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{(t_k - t_{k-1})^2}{2} \\ t_k - t_{k-1} \end{bmatrix} v_{k-1}$$

Time difference squared/2 multiplied by acceleration error between t_k and t_{k-1} gives a position error

Acceleration error from t_{k-1} to t_k

Time difference multiplied by acceleration error between t_k and t_{k-1} gives a velocity error

Measurement model:

$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} + w_k = x_k + w_k = HX_k + w_k$$

Sensor measures only position at time t_k

Measurement errors in position at time t_k

where w_k **is zero-mean** $\left(E[w_k]=0\right)$ **with variance** σ_w^2 ← Derived from sensor signal-to-noise ratio

v_k **is zero-mean** $\left(E[v_k]=0\right)$ **with variance** σ_{ncv}^2 ← Design parameter **How do we pick it?**

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NCV Filter Design for Filtered Position

Selecting the Process Noise Variance [12]

- Given

$$\Gamma_D^2 = \frac{T^4 A_{\max}^2}{\sigma_w^2} \qquad \Gamma_{DWNA}^2 = \frac{T^4 \sigma_{ncv}^2}{\sigma_w^2} = \frac{\beta^2}{1-\alpha}$$

- Let

$$\kappa_1^{pos} = \frac{\Gamma_{DWNA}}{\Gamma_D} = \frac{\sigma_{ncv}}{A_{\max}} \Rightarrow \sigma_{ncv} = \kappa_1^{pos} A_{\max} \Rightarrow \kappa_1(\Gamma_D)$$

- For $MinMaxMSE^{pos}$ for a sustained maneuver

$$\kappa_1^{pos,\max}(\Gamma_D) = 1.69(0.66)^{\bar{\Gamma}_D}(1.03)^{\bar{\Gamma}_D^2}, \; 0.001 \le \Gamma_D \le 10, \quad \text{where} \quad \bar{\Gamma}_D = \log 10(\Gamma_D)$$

- Subject to the minimum

$$\kappa_1^{pos,\min}(\Gamma_D) = 0.87(0.90)^{\bar{\Gamma}_D}(0.97)^{\bar{\Gamma}_D^2}, \; 0.001 \le \Gamma_D \le 10$$

NCV Kalman Filter: Selecting the Process Noise Variance [12]

$A_{\max}$ = maximum acceleration of the target

σ_w = standard deviation of measurement errors

T = measurement period

$$\Gamma_D = \frac{T^2 A_{\max}}{\sigma_w}$$

Example:

$A_{\max} = 40\ \text{m/s}^2$

$\sigma_w = 120\ \text{m}$

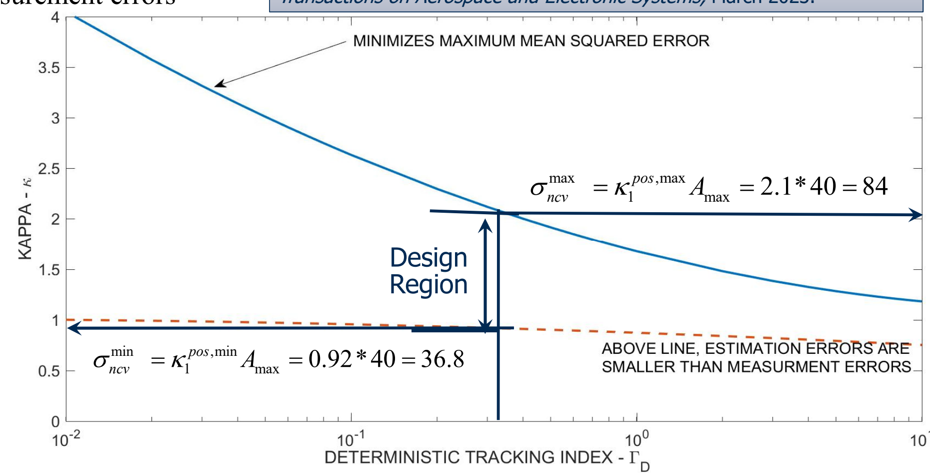
$T = 1\ \text{s}$

$$\Gamma_D = \frac{1^2(40)}{120} = 0.33$$

$$\sigma_{ncv} = \kappa_1^{pos} A_{\max}$$

How do we pick κ_1^{pos}?

W. D. Blair and Y. Bar-Shalom, "MSE Design of Nearly Constant Velocity Kalman Filters for Tracking Targets with Deterministic Maneuvers," *IEEE Transactions on Aerospace and Electronic Systems,* March 2023.



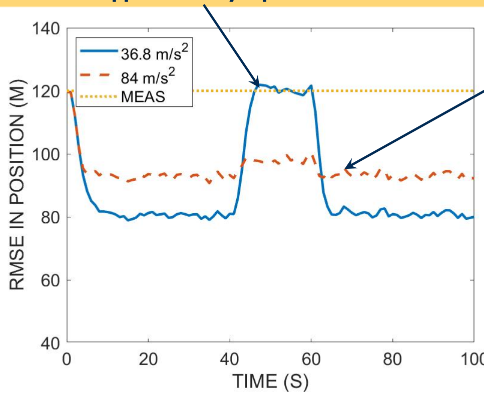
W. D. Blair, "Design of Nearly Constant Velocity Filter for Brief Maneuvers," *Proceedings of the 2011 International Conference on Information Fusion,* Chicago, IL, July 2011

Computer Simulations: Example 1

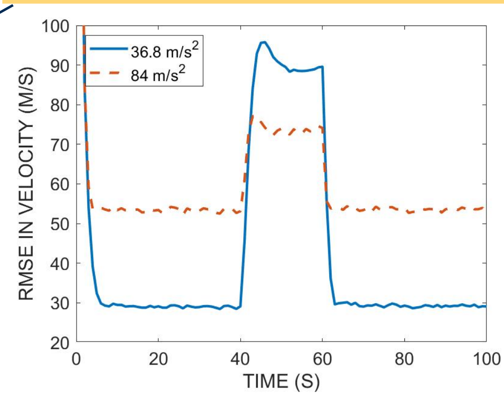
Consider Monte Carlo Simulations of target that maneuvers at 40 m/s^2 from 40 to 60 s.

Smaller process noise variance gives errors for maximum maneuver that approximately equal to the measurement errors.

Larger process noise variance minimizes the average maximum mean squared errors during maximum maneuver.



Position Errors



Velocity Errors

NCV Radar Tracking with LFM Waveforms

For radar range measured with an LFM waveform, the measured range is given by

$$z_k = r_k + \Delta t\ \dot{r}_k + w_k$$

where

$$\Delta t = \frac{f_0\,\tau}{f_1 - f_0}$$

$f_0 =$ initial carrier frequency $\quad \tau =$ waveform duration

$f_1 =$ final carrier frequency

System is defined by

$$x_k = \begin{bmatrix} r_k & \dot{r}_k \end{bmatrix}^T$$

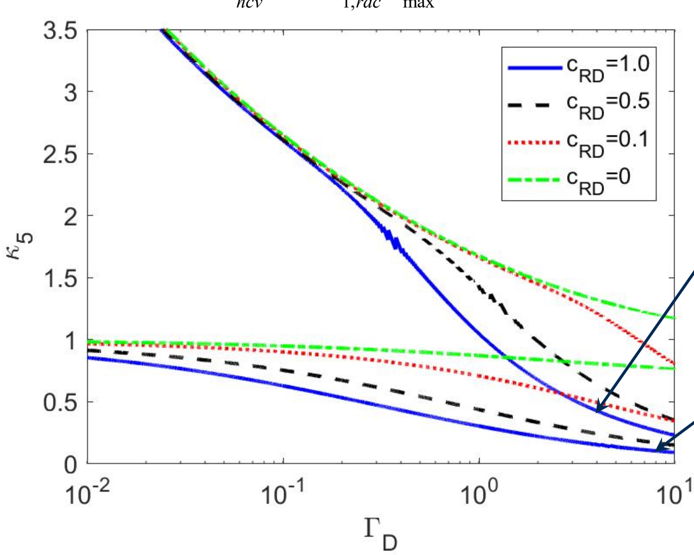
$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} \frac{T^2}{2} & T \end{bmatrix}^T \qquad Q = \sigma_v^2$$

$$H = \begin{bmatrix} 1 & \Delta t \end{bmatrix} \qquad R = \sigma_r^2 = \sigma_w^2 \qquad \Gamma_{DWNA} = \frac{T^2 \sigma_{ncv}^2}{\sigma_w^2} \qquad c_{RD} = \frac{\Delta t}{T} \qquad \Gamma_D = \frac{T^2 A_{max}}{\sigma_w^2}$$

According to [6,7,8], filter performance is specified by Γ_{DWNA} and c_{RD}.

NCV Radar Tracking with LFM Waveforms [6]

$$\sigma_{ncv} = \kappa_{1,rdc} A_{max}$$



Positive c_{RD} allows for smaller process noise variance for MinMaxMSE^{pos}

Positive c_{RD} allows for smaller overall process noise variance

NCV Radar Tracking with LFM Waveforms

Selecting the Process Noise Variance

$$\sigma_{ncv} = \kappa_{1,rdc}^{pos} A_{\max}$$

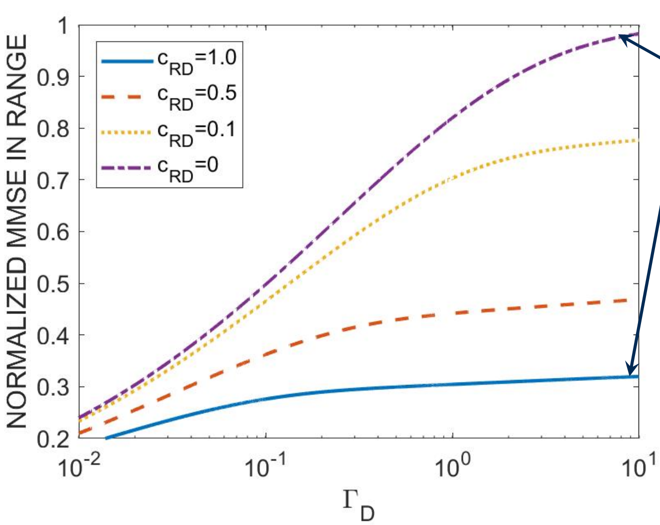
$$\kappa_{5,rdc}^{pos,\max}(\Gamma_D, c_{RD}) = a_0^{\max} (a_1^{\max})^{\bar{\Gamma}_D} (a_2^{\max})^{\bar{\Gamma}_D^2}$$

where $\bar{\Gamma}_D = \log(\Gamma_D)$

$$\kappa_{5,rdc}^{pos,\min}(\Gamma_D, c_{RD}) = a_0^{\min} (a_1^{\min})^{\bar{\Gamma}_D} (a_2^{\min})^{\bar{\Gamma}_D^2}$$

c_{RD}		a_0	a_1	a_2
1.0	$\kappa_{5,rdc=1}^{pos,\max}$	1.05	0.29	0.73
1.0	$\kappa_{5,rdc=1}^{pos,\min}$	0.30	0.38	0.79
0.5	$\kappa_{5,rdc=0.5}^{pos,\max}$	1.33	0.36	0.75
0.5	$\kappa_{5,rdc=0.5}^{pos,\min}$	0.43	0.44	0.79
0.1	$\kappa_{5,rdc=0.1}^{pos,\max}$	1.64	0.57	0.94
0.1	$\kappa_{5,rdc=0.1}^{pos,\min}$	0.69	0.62	0.84

NCV Radar Tracking with LFM Waveforms



Positive $c_{RD} = 1.0$ allows for much smaller $MinMaxMSE^{pos}$

For highly maneuvering targets, LFM waveforms are required to achieve error reduction in range.

Error reduction due to the use of an LFM waveform is in range only.

Mode estimation in the IMM Estimator in three dimensions is meaningfully improved by the use of LFM waveforms.

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Nearly Constant Acceleration (NCA) Motion Model

Motion model:

$$X_k = \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} = F_{k-1}X_{k-1} + G_{k-1}v_{k-1} = \begin{bmatrix} 1 & t_k - t_{k-1} & \frac{(t_k - t_{k-1})^2}{2} \\ 0 & 1 & t_k - t_{k-1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \dot{x}_{k-1} \\ \ddot{x}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{(t_k - t_{k-1})^2}{2} \\ t_k - t_{k-1} \\ 1 \end{bmatrix} v_{k-1}$$

Acceleration error from t_{k-1} to t_k

Measurement model:

$$z_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} + w_k = x_k + w_k = HX_k + w_k$$

where w_k is zero-mean $(E[w_k]=0)$ with variance σ_w^2

v_k is zero-mean $(E[v_k]=0)$ with variance σ_{nca}^2 ← Design parameter — How do we pick it?

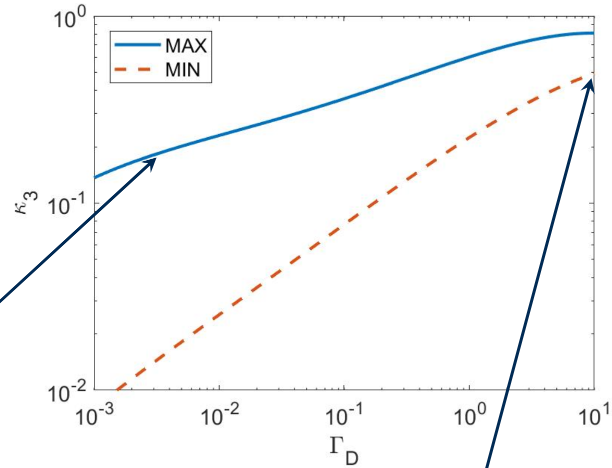
Design of NCA Track Filter [9]

$$\sigma_{nca} = \kappa_3 A_{\max}$$

$$\kappa_3^{\max}(\Gamma_D) = 0.6(1.62)^{\bar{\Gamma}_D}(0.921)^{\bar{\Gamma}_D^2}(0.922)^{\bar{\Gamma}_D^3}\ (0.983)^{\bar{\Gamma}_D^4}$$

where $\bar{\Gamma}_D = \log(\Gamma_D)$

$$\kappa_3^{\min}(\Gamma_D) = 0.223(2.69)^{\bar{\Gamma}_D}(0.877)^{\bar{\Gamma}_D^2}(0.941)^{\bar{\Gamma}_D^3}$$



Process noise should always be less than the maximum acceleration and that fraction decreases as the Γ_D decreases.

As a rule of thumb, picking $\sigma_{nca} = 0.5A_{\max}$ will ensure that the estimation errors are not worse than the measurement errors.

Design of NCA Track Filter [9]

$$\sigma_{nca} = \kappa_3 A_{\max}$$

$$\kappa_3^{\max}(\Gamma_D) = 0.6(1.62)^{\bar{\Gamma}_D}(0.921)^{\bar{\Gamma}_D^2}(0.922)^{\bar{\Gamma}_D^3}\ (0.983)^{\bar{\Gamma}_D^4}, \quad \text{where } \bar{\Gamma}_D = \log 10(\Gamma_D)$$

$$\kappa_3^{\min}(\Gamma_D) = 0.223(2.69)^{\bar{\Gamma}_D}(0.877)^{\bar{\Gamma}_D^2}(0.941)^{\bar{\Gamma}_D^3}$$

Example:

$A_{\max} = 40$ m/s^2

$\sigma_w = 120$ m

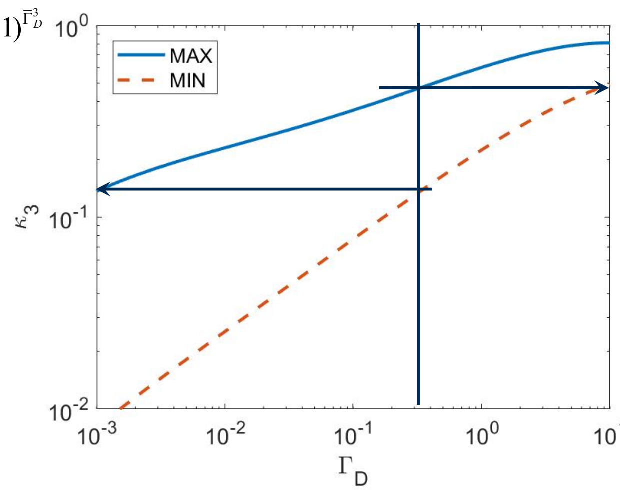
$T = 1$ s

$$\Gamma_D = \frac{1^2(40)}{120} = \frac{1}{3}$$

$\sigma_{nca} = \kappa_3\ A_{\max}$

$\kappa_3^{\max}(\Gamma_D) = 0.475 \rightarrow \sigma_{nca}^{\max} = 19$ m/s^2

$\kappa_3^{\min}(\Gamma_D) = 0.135 \rightarrow \sigma_{nca}^{\min} = 5.4$ m/s^2

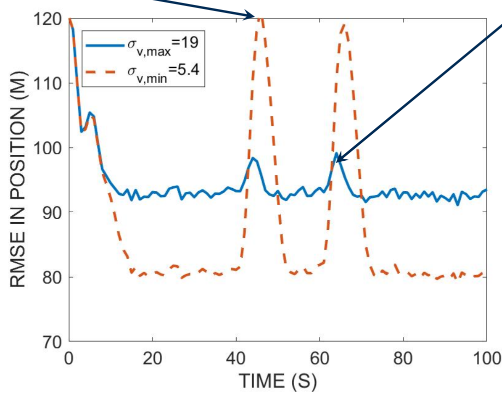


Example 2: Tracking with NCA Filter

Consider Monte Carlo Simulations of target that maneuvers at 40 m/s² from 40 to 60 s.

Smaller process noise variance gives errors for maximum maneuver that approximately equal to the measurement errors.

Larger process noise variance minimizes the maximum mean squared errors during maximum maneuver.



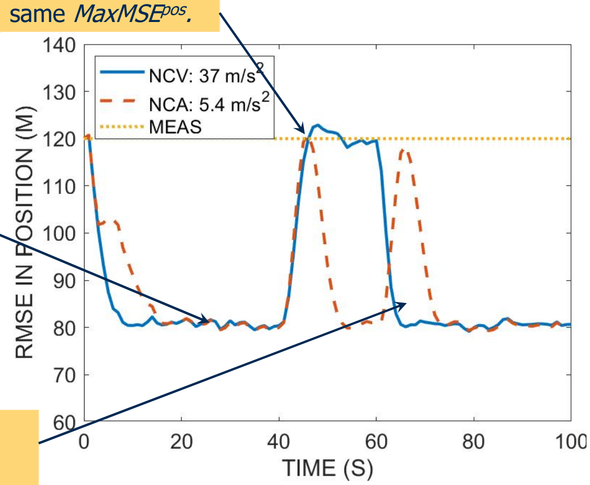
NCV Filter Versus NCA Filter

$$MaxMSE^{pos} \le \sigma_w^2 \text{ Design Criteria}$$

Consider Monte Carlo Simulations of target that maneuvers at 40 m/s² from 40 to 60 s.

Both the NCV and NCA filters have essentially the same *MaxMSE^pos^*.

Both the NCV and NCA filters have the same errors when the target is not maneuvering.



Note that degraded tracking persists after the maneuver ends.

NCV Filter Versus NCA Filter

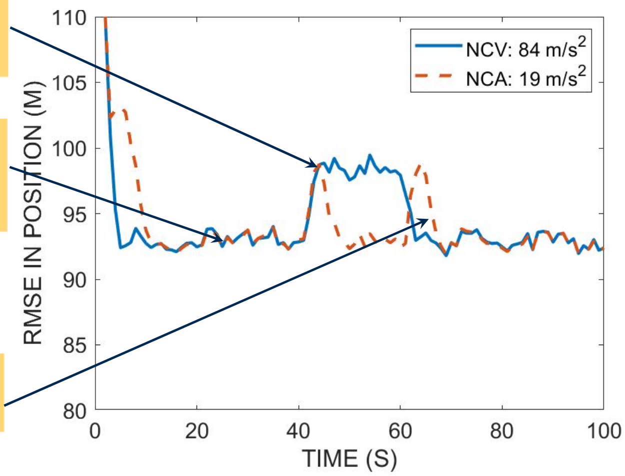
$MinMaxMSE^{pos}$ Design Criteria

Consider Monte Carlo Simulations of target that maneuvers at 40 m/s² from 40 to 60 s.

Both the NCV and NCA filters have the same $MaxMSE^{pos}$

Both the NCV and NCA filters have the same errors when the target is not maneuvering.

Note that degraded tracking persists after the maneuver ends.



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Track Filtering: Radar Measurements

Radar Measurements: Spherical Coordinates

Measurement at time k is given by

$$Z_k = \begin{bmatrix} r_k \\ a_k \\ e_k \end{bmatrix} = h_k\left(X_k\right) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2 + z_k^2} \\ \tan^{-1}\left(\frac{y_k}{x_k}\right) \\ \tan^{-1}\left(\frac{z_k}{\sqrt{x_k^2 + y_k^2}}\right) \end{bmatrix}$$

where:

r_k = measurement of target range

a_k = measurement of target azimuth

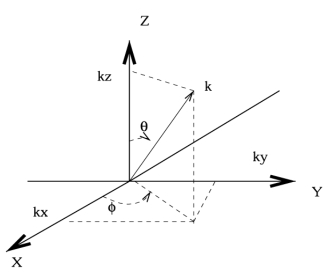
e_k = measurement of target elevation

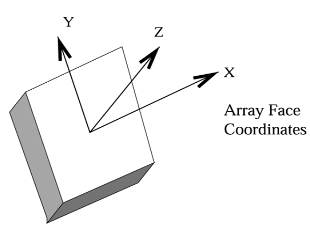
$\left(x_k, y_k, z_k\right)$ = $\left(x, y, z\right)$ coordinates of the target position

X_k = target state vector = $\left[x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k \quad z_k \quad \dot{z}_k\right]^T$

Track Filtering: Radar Measurements

Sine Space 3-D





		Array Face Coordinates	**Sine Space**
$\sin\left(\theta_{Az}\right)$	$=\sin\left(\theta\right)\cos\left(\phi\right)$	$=\frac{x}{r}$	$=k_x$
$\sin\left(\theta_{El}\right)$	$=\sin\left(\theta\right)\cos\left(\phi\right)$	$=\frac{y}{r}$	$=k_y$
	$\cos\left(\theta\right)$	$=\frac{z}{r}$	$=k_z$
		$\sqrt{x^2+y^2+z^2}$	$=r$

Extended Kalman Filter (EKF) for Radar Tracking

Consider the nonlinear system state

$$X_k = F_{k-1} X_{k-1} + G_{k-1} v_{k-1}$$

with observations

$$Z_k = h_k\left(X_k\right) + w_k$$

where: X_k = System State

v_k = White Gaussian errors for system state process with $v_k \sim N\left(0, Q_k\right)$

w_k = White Gaussian errors in the measurements process with $w_k \sim N\left(0, R_k\right)$

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Track Filtering: Extended Kalman Filter (EKF) for Radar Tracking

Algorithm

Time Update:

$$X_{k|k-1} = F_{k-1} X_{k-1|k-1}$$

$$P_{k-1|k} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + G_{k-1} Q_{k-1} G_{k-1}^T$$

Measurement Update:

$$K_k = P_{k|k-1} H_k^T [H_k P_{k|k-1} H_k^T + R_k]^{-1}$$

$$X_{k|k} = X_{k|k-1} + K_k [Z_k - h_k(X_{k|k-1})]$$

$$P_{k|k} = [I - K_k H_k] P_{k|k-1} = P_{k|k-1} - K_k H_k P_{k|k-1}$$

where:

$$H_k = \left.\frac{\partial h_k(X_k)}{\partial X_k}\right|_{X_k = X_{k|k-1}}$$

Filter Design of EKF for Radar Tracking

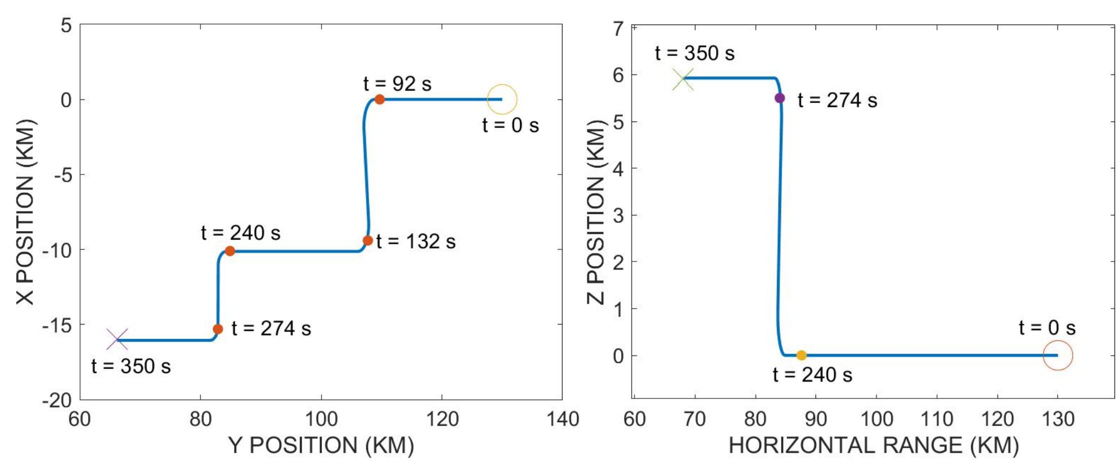
- Radar measurement errors are approximately stationary in range and angle coordinates
- Range measurement errors are nearly stationary in crossrange with the variance smoothly change with range of the target.
- Track filter design for radar tracking is performed in range and crossrange
  - Range only track filter designed using scalar methods for supporting signal processing.

$$\text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r \sqrt{3}}$$

  - 3D (*i.e.,* Cartesian x, y, z) track filter is designed using variance of crossrange errors for selection of the process noise variance versus range.

$$3D \text{ Filter}: \quad \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}$$

Scenario 1 – Target Trajectory



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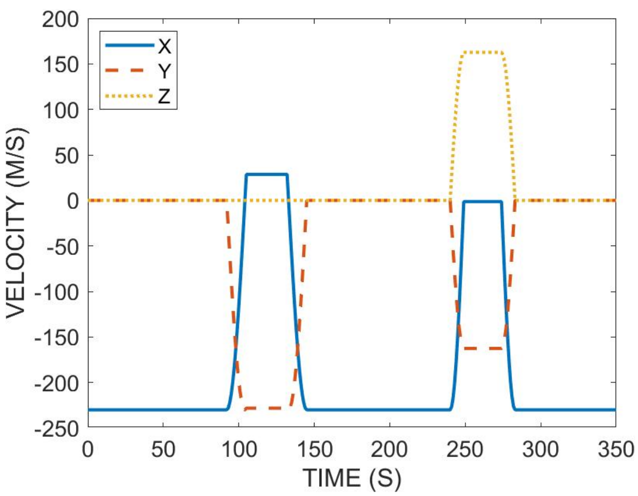
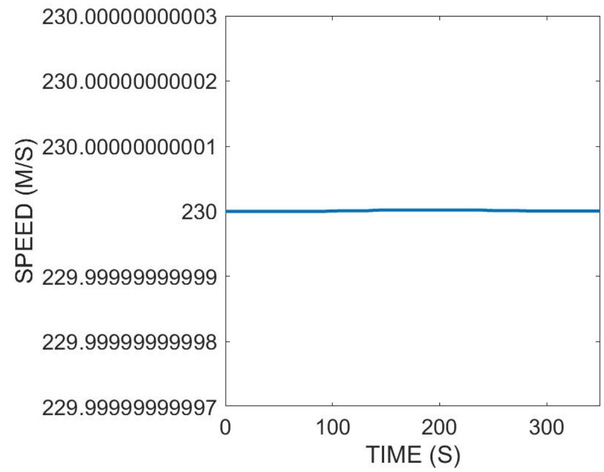
Scenario 1 – Target Trajectory





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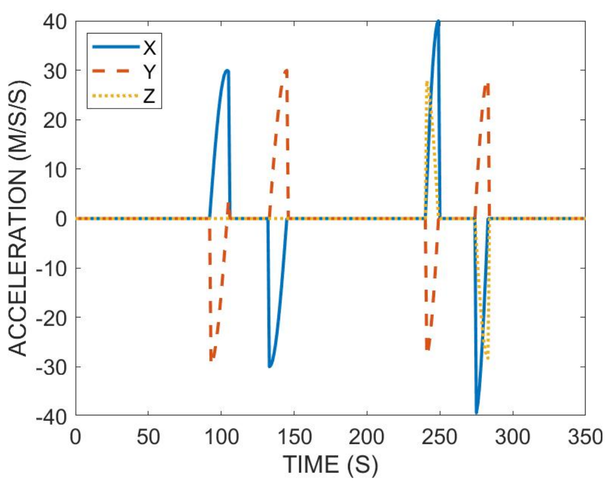
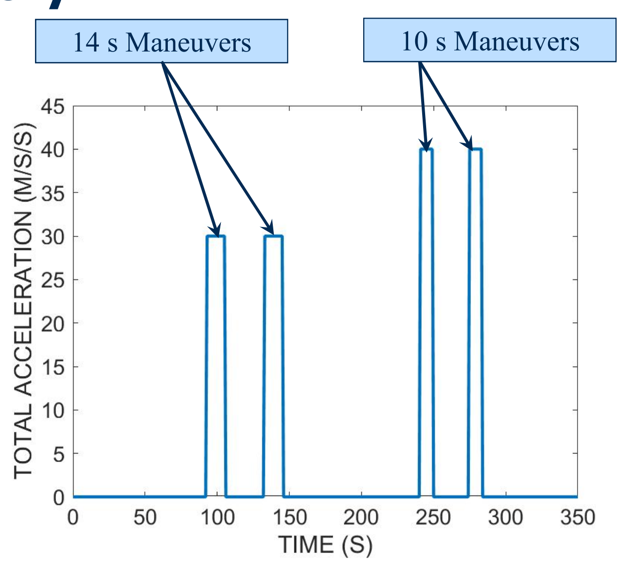
Scenario 1 – Target Trajectory





Example 3: Two NCV Filters with DWNA

- Monotone Monopulse Radar – Minimum Process Noise Variance (Sim 1)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCV Filter with Discrete White Noise Acceleration (DWNA)

$$\sigma_{ncv}^{\min} = \kappa_1^{pos,\min} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_1^{pos,\min}(\Gamma_D) = 0.87(0.9)^{\bar{\Gamma}_D}(0.98)^{\bar{\Gamma}_D^2}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter: } \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r \sqrt{3}}$$

- Monotone Monopulse Radar - $MinMaxMSE^{pos}$ (Sim 2)

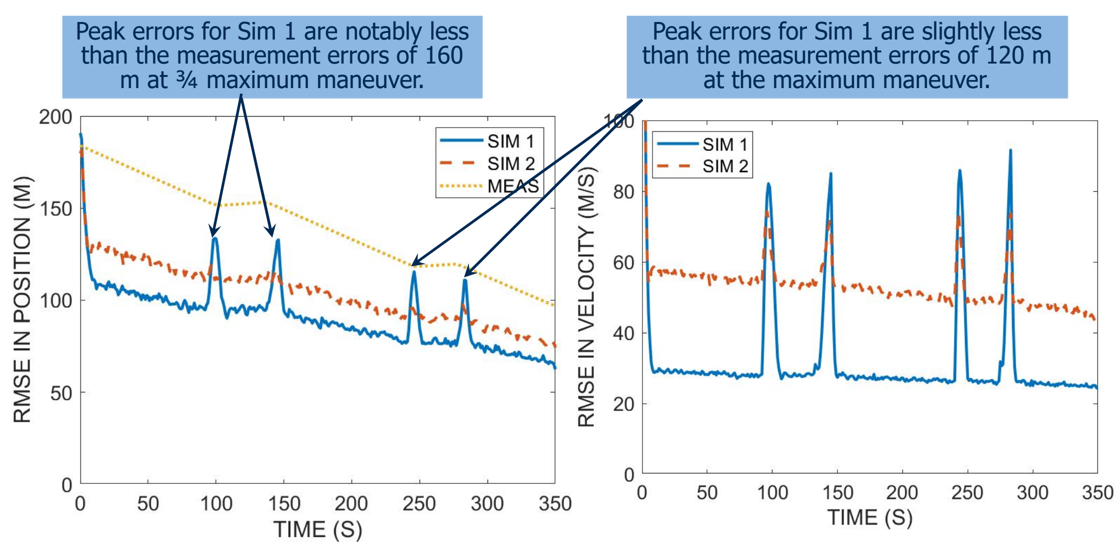
$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCV Filter with DWNA

$$\sigma_{ncv}^{\max} = \kappa_1^{pos,\max} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_1^{pos,\max}(\Gamma_D) = 1.69(0.66)^{\bar{\Gamma}_D}(1.03)^{\bar{\Gamma}_D^2}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter: } \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r \sqrt{3}}$$

Example 3: Two NCV Filters with DWNA



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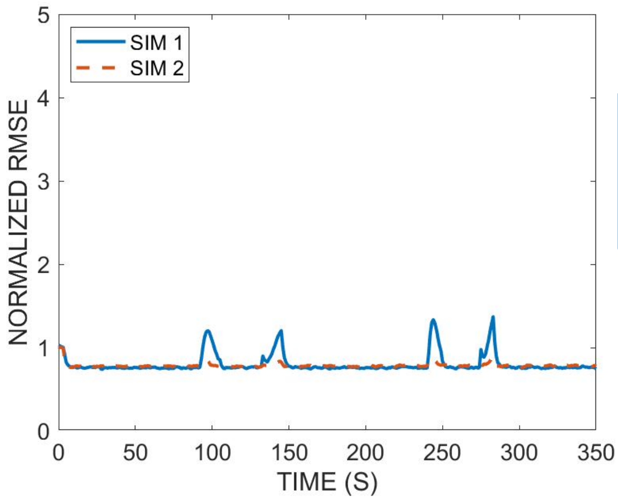
Example 3: Two NCV Filters with DWNA



Normalized RMSE for position and velocity shows that the state covariance is too large except when maneuvers are present and the smaller process noise variance in Sim 1.

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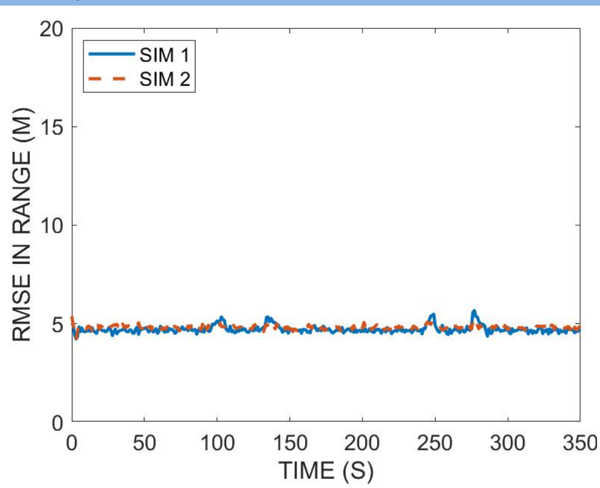
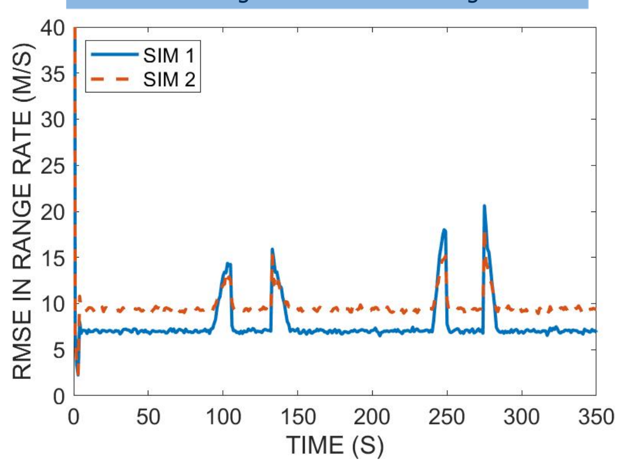
Example 3: Two NCV Filters with DWNA

MinMaxMSE^{pos} is the measurement error in range. No NCV filter will provide error reduction for a monotone waveform.

Note that the peak errors are similar for both cases, but the errors for Sim 1 are less when the target is not maneuvering.





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Example 4: Two NCV Filters for Monotone Versus LFM

- Monotone Monopulse Radar – $MinMaxMSE^{pos}$ (Sim 1)

  $\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

  NCV Filter with Discrete White Noise Acceleration (DWNA)

  $$\sigma_{ncv}^{\max} = \kappa_1^{pos,\max}\frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_1^{pos,\max}(\Gamma_D) = 1.69(0.66)^{\overline{\Gamma}_D}(01.03)^{\overline{\Gamma}_D^2}, \quad \overline{\Gamma}_D = \log10(\Gamma_D)$$

  $$3D \text{ Filter}: \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az},\sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r\sqrt{3}}$$

- LFM Monopulse Radar - $MinMaxMSE^{pos}$ (Sim 2)

  $\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

  Range-Doppler-Coupling Coefficient (rdc) = 1.0

  NCV Filter With DWNA

  $$\text{Range: } \sigma_{ncv,\text{rdc}=1}^{\max} = \kappa_{5,\text{rdc}=1}^{pos,\max}\frac{A_{\max}}{\sqrt{3}}$$

  $$\text{where } \kappa_{5,\text{rdc}=1}^{pos,\max}(\Gamma_D) = 1.05(0.29)^{\overline{\Gamma}_D}(0.73)^{\overline{\Gamma}_D^2}, \quad \overline{\Gamma}_D = \log10(\Gamma_D), \quad \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r\sqrt{3}}$$

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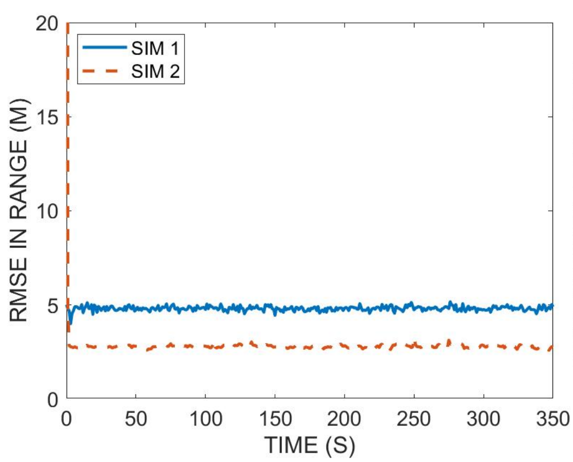
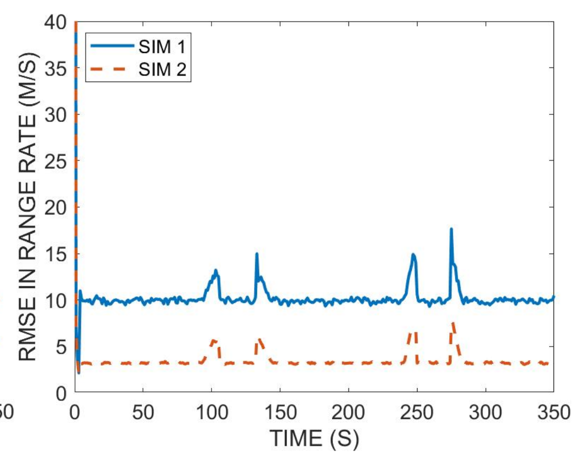
Example 4: Two NCV Filter Designs for Monotone Versus LFM

MinMaxMSE^pos is the measurement error in range for the NCV Kalman filter with monotone waveform. Measurement error is reduced with LFM waveforms.





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Example 5: Two NCA Filters with DWPA

- Monotone Monopulse Radar - Minimum Process Noise Variance (Sim 1)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCA Filter With Discete Wiener Process Acceleration

$$\sigma_{nca}^{min} = \kappa_{1,\infty}^{min}\frac{A_{max}}{\sqrt{3}} \text{ where } \kappa_{1,\infty}^{min}(\Gamma_D) = 0.223(2.69)^{\bar{\Gamma}_D}(0.877)^{\bar{\Gamma}_D^2}(0.941)^{\bar{\Gamma}_D^2}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \Gamma_D = \frac{T^2 A_{max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{max}}{\sqrt{3}\sigma_r}$$

- Monotone Monopulse Radar - *MinMaxMSE*^{pos} (Sim 2)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCA Filter With Discete Wiener Process Acceleration

$$\sigma_{nca}^{max} = \kappa_{3}^{max}\frac{A_{max}}{\sqrt{3}} \text{ where } \kappa_{3}^{max}(\Gamma_D) = 0.6(1.62)^{\bar{\Gamma}_D}(0.921)^{\bar{\Gamma}_D^2}(0.922)^{\bar{\Gamma}_D^3}(0.983)^{\bar{\Gamma}_D^4}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \Gamma_D = \frac{T^2 A_{max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{max}}{\sqrt{3}\sigma_r}$$

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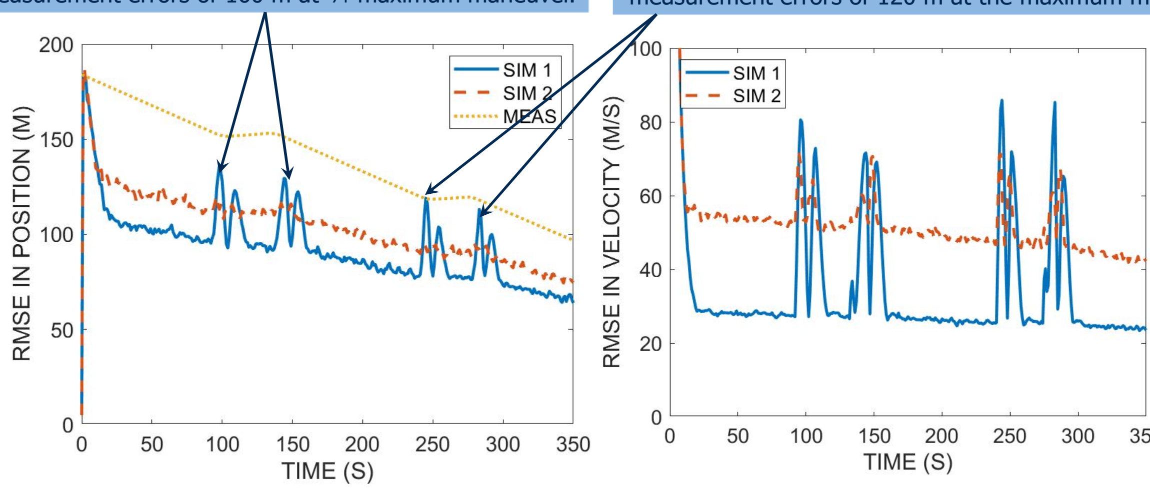
Example 5: Two NCA Filter Designs with DWPA

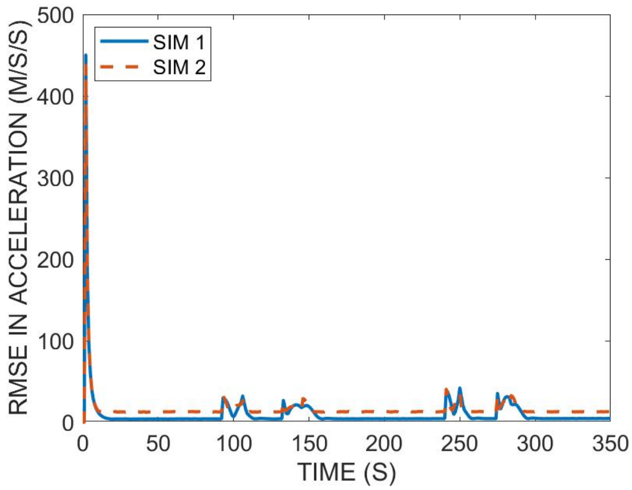
Peak errors for Sim 1 are notably less than the measurement errors of 160 m at ¾ maximum maneuver.

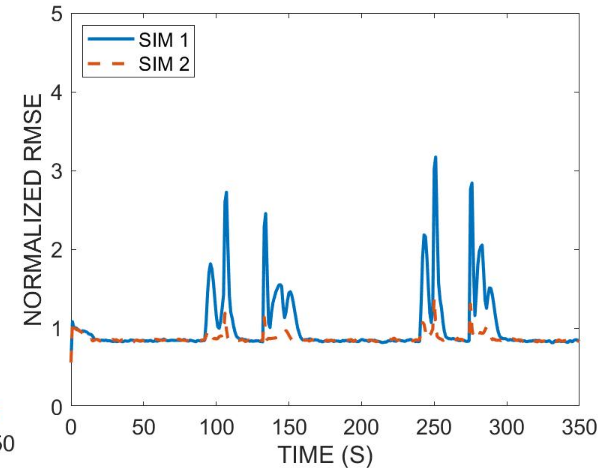
Peak errors for Sim 1 are slightly less than the measurement errors of 120 m at the maximum maneuver.



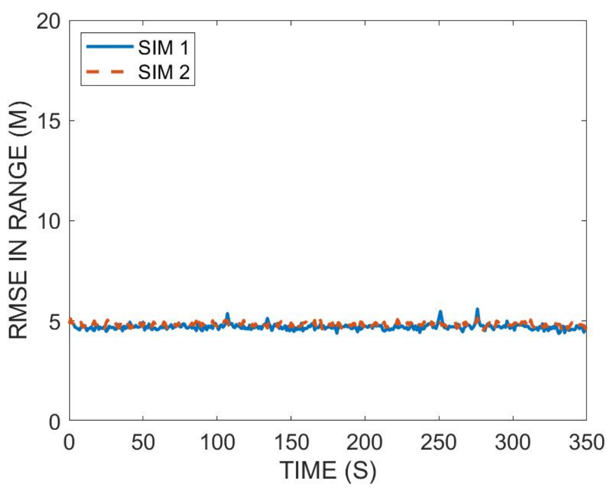
Example 5: Two NCA Filter Designs with DWPA

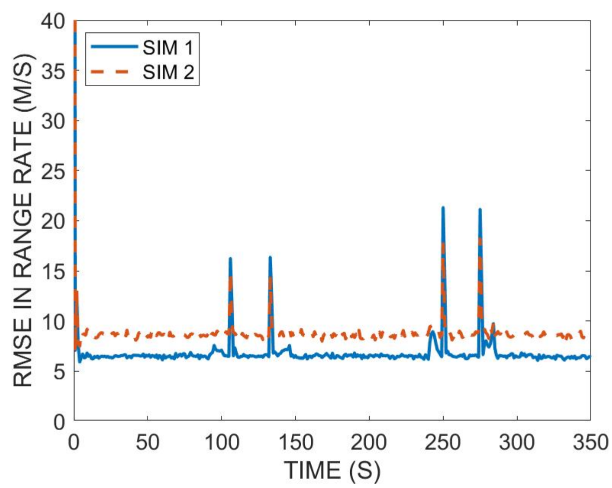
Normalized RMSE for position and velocity shows that the state covariance is too large except when maneuvers are present. The smaller process noise variance in Sim 1 gives poor inconsistency during maneuvers



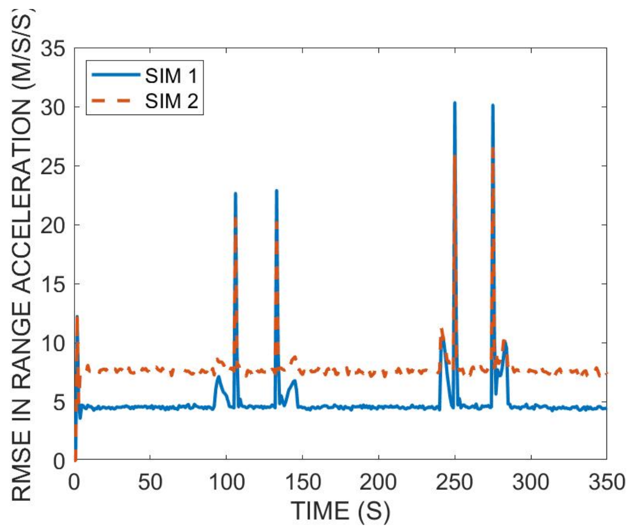


Example 5: Two NCA Filter Designs with DWPA





Example 5: Two NCA Filter Designs with DWPA



Outline

- Introduction
- Overview of Target Track Filtering
- Nearly-Constant Velocity (NCV) Track Filter Design
- Nearly Constant Acceleration (NCA) Track Filter Design
- Track Filter Design for Radar Tracking of Maneuvering Targets
- **Filter Design and IMM Estimator**
- Concluding Remarks

Multiple Model Tracking of Maneuvering Targets

Pointer to one of N models

Consider the system state

$$X_{k+1} = F_k\left(\theta_{k+1}\right) X_k + G_k\left(\theta_{k+1}\right) v_k$$

Example: N = 2 models

$\theta_{k+1}=1 \Rightarrow$ Constant Velocity

$\theta_{k+1}=2 \Rightarrow$ Accelerating

θ_{k+1} is finite state Markov Chain with p_{ij} probability of switching from model i to model j

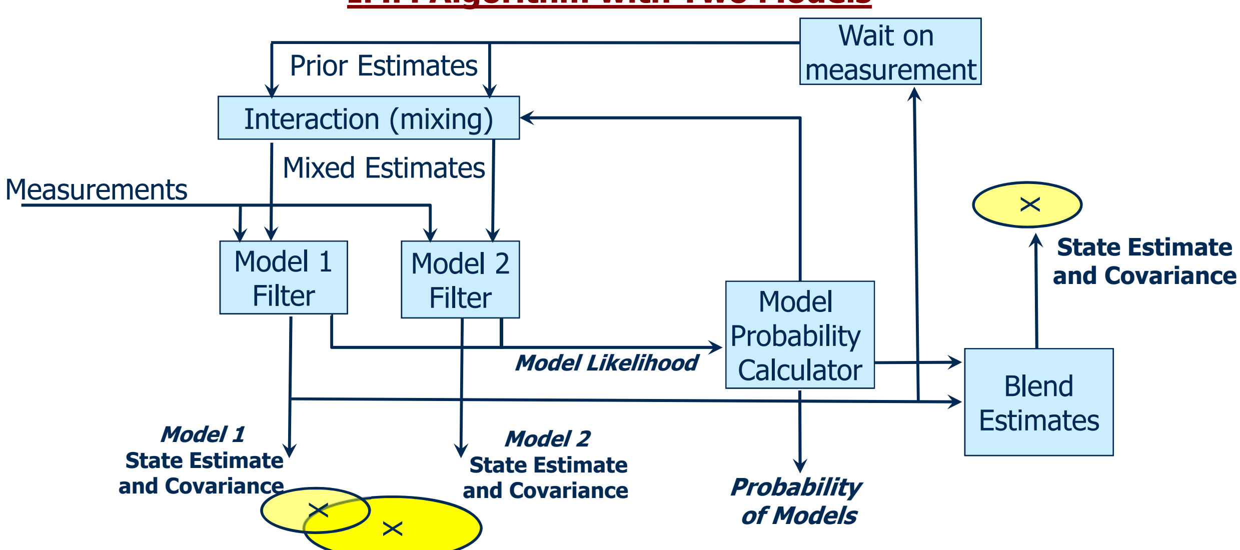
with measurements

$$Z_k = H_k\left(\theta_k\right) X_k + w_k$$

This model for maneuvering targets **does not** lead to multiple Kalman filters operating in parallel and picking the best!!!

Interacting Multiple Model (IMM) Estimator

IMM Algorithm with Two Models



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Example 6: NCV Versus IMM CVCA

- Monopulse Radar – Minimum Process Noise Variance Design of NCV (Sim 1)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCV Filter with Discrete White Noise Acceleration (DWNA)

$$\sigma_{ncv}^{\min} = \kappa_1^{pos,\min} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_1^{pos,\min}(\Gamma_D) = 0.87(0.9)^{\bar{\Gamma}_D}(0.98)^{\bar{\Gamma}_D^2}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \ \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az},\sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r\sqrt{3}}$$

- Monopulse Radar - IMM CVCA with Minimum Process Noise Variance Design for NCA Filter (Sim 2)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

Model 1: NCV Kalman filter with $\sigma_{ncv}^{\min} = 1$ m/s^2

Model 2: NCA Filter With Discete Wiener Process Acceleration

$$\sigma_{nca}^{\min} = \kappa_3^{\min} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_3^{\min}(\Gamma_D) = 0.223(2.69)^{\bar{\Gamma}_D}(0.877)^{\bar{\Gamma}_D^2}(0.941)^{\bar{\Gamma}_D^3}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \ \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az},\sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sqrt{3}\sigma_r}$$

$$p_{11} = 0.9 + 0.1\exp(-T/2.0), \ p_{12} = 1 - p_{11}; \ p_{22} = 0.8 + 0.2\exp(-T/2.0), \ p_{21} = 1 - p_{22}$$

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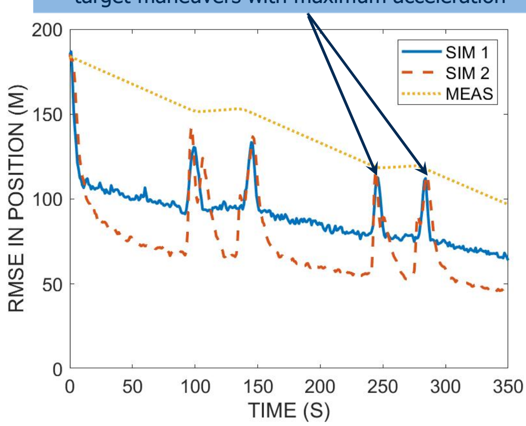
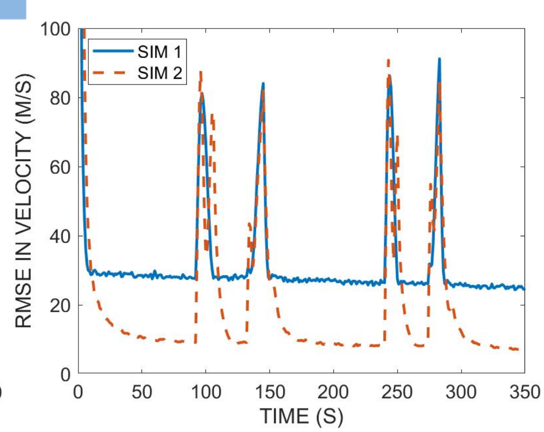
Example 6: NCV Versus IMM CVCA

Peak errors approach the measurement errors when target maneuvers with maximum acceleration





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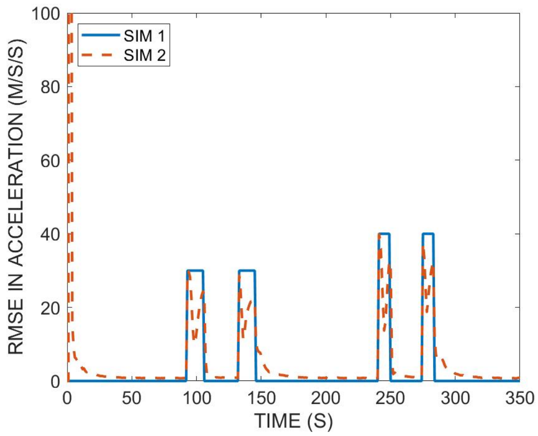
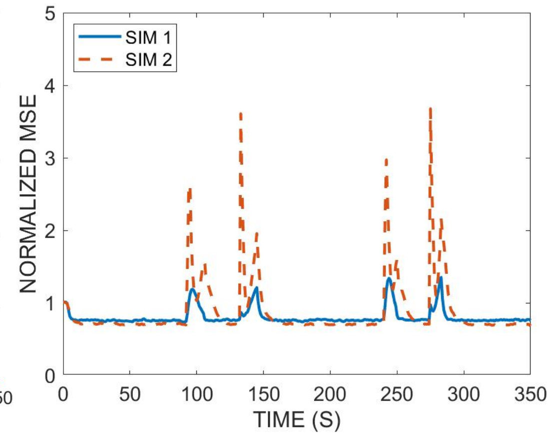
Example 6: NCV Versus IMM CVCA

Normalized RMSE for position and velocity shows that the state covariance is too large except when maneuvers are present in Sim 2.





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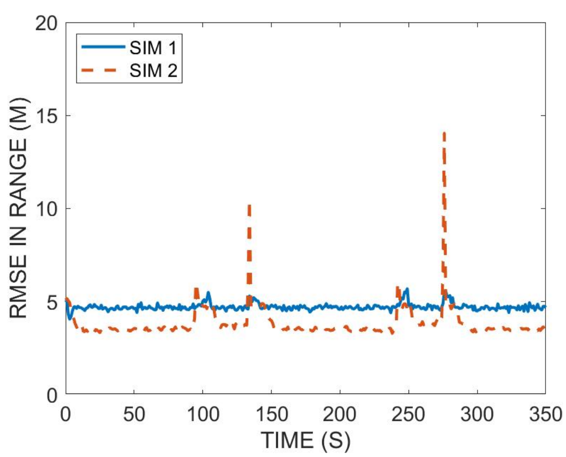
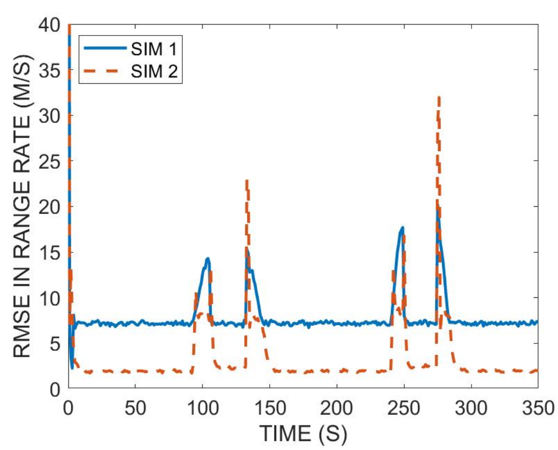
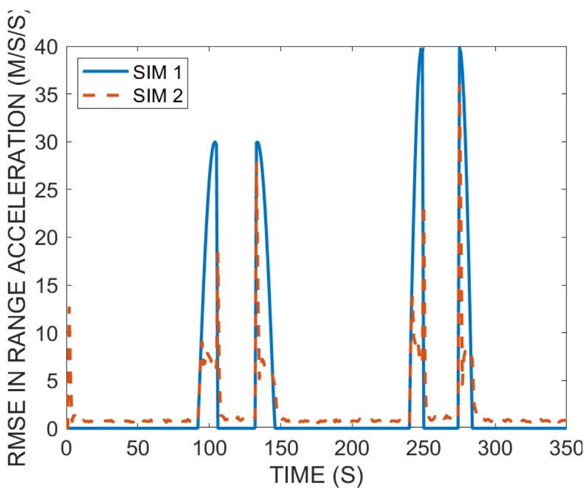
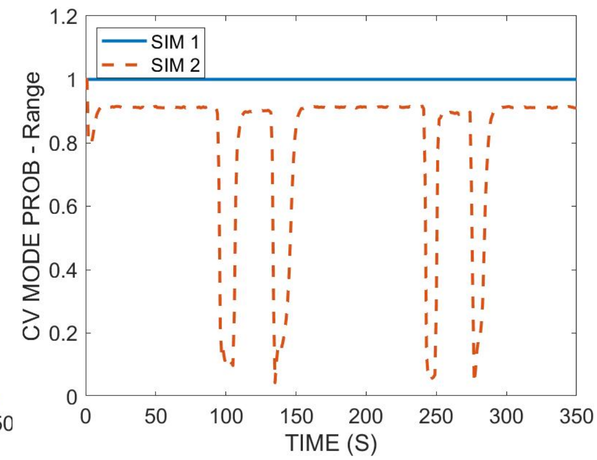




Example 6: NCV Versus IMM CVCA





Example 7: NCV Versus IMM CVCA

- Monopulse Radar – *MinMaxMSE*^{pos} NCV Filter Design (Sim 1)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

NCV Filter with DWNA

$$\sigma_{ncv}^{\max} = \kappa_1^{pos,\max} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_1^{pos,\max}(\Gamma_D) = 1.7(0.66)^{\bar{\Gamma}_D}(1.02)^{\bar{\Gamma}_D^2}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \ \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sigma_r \sqrt{3}}$$

- Monopulse Radar - IMM CVCA with *MinMaxMSE*^{pos} NCA Filter Design (Sim 2)

$\sigma_r = 5$ m, $\sigma_{az} = 1$ mrad, $\sigma_{el} = 1$ mrad, $T = 1$ s

Model 1: NCV Kalman filter with $\sigma_{ncv}^{\min} = 1$ m/s^2

Model 2: NCA Filter With Discete Wiener Process Acceleration

$$\sigma_{nca}^{\max} = \kappa_3^{\max} \frac{A_{\max}}{\sqrt{3}} \text{ where } \kappa_3^{\max}(\Gamma_D) = 0.6(1.62)^{\bar{\Gamma}_D}(0.921)^{\bar{\Gamma}_D^2}(0.922)^{\bar{\Gamma}_D^3}(0.983)^{\bar{\Gamma}_D^4}, \quad \bar{\Gamma}_D = \log10(\Gamma_D)$$

$$3D \text{ Filter}: \ \Gamma_D = \frac{T^2 A_{\max}}{r\sqrt{3}\max\{\sigma_{az}, \sigma_{el}\}}, \quad \text{Range Filter: } \Gamma_D = \frac{T^2 A_{\max}}{\sqrt{3}\sigma_r}$$

$$p_{11} = 0.9 + 0.1\exp(-T/2.0), \ p_{12} = 1 - p_{11}; \ p_{22} = 0.8 + 0.2\exp(-T/2.0), \ p_{21} = 1 - p_{22}$$

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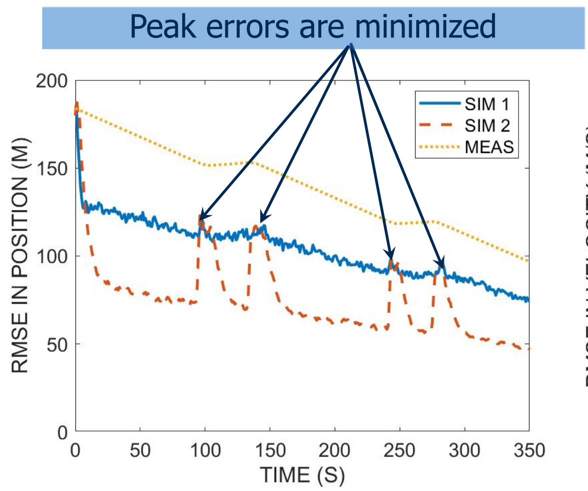
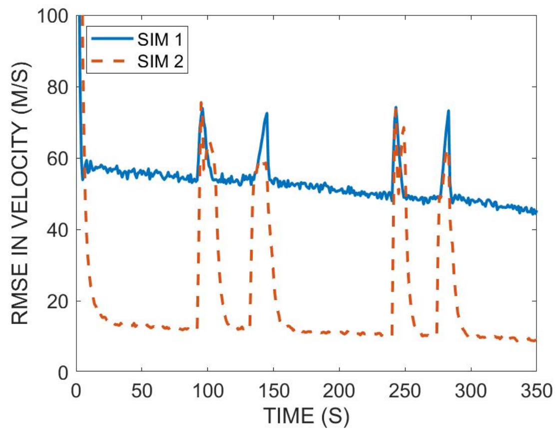
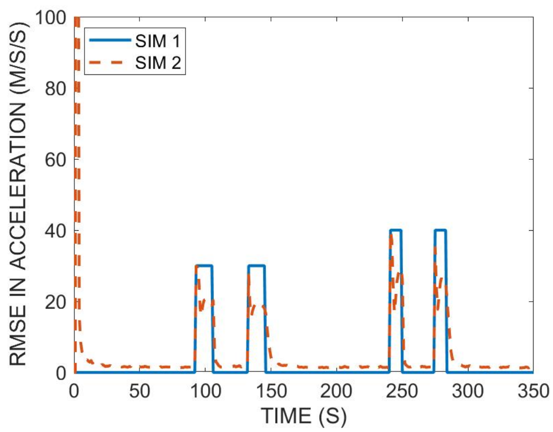
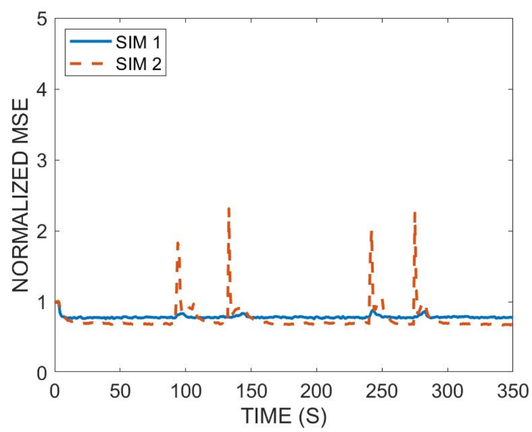




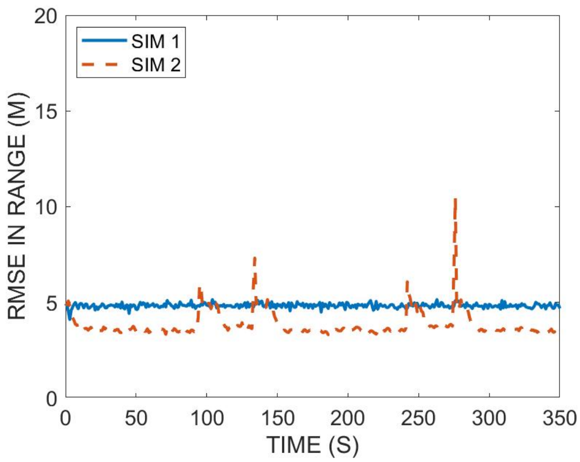
Example 7: NCV Versus IMM CVCA

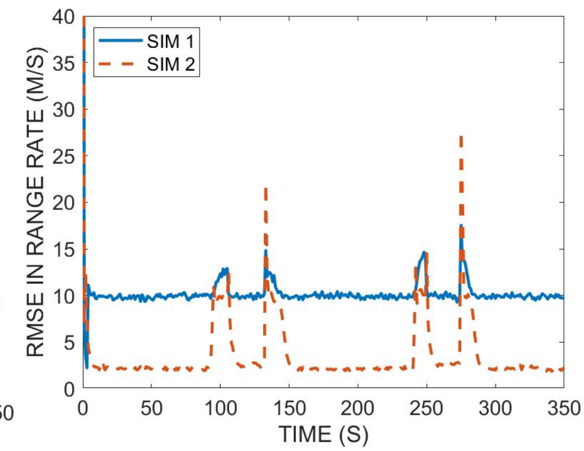
Normalized MSE for position and velocity shows that the state covariance is too large except when maneuvers are present in Sim 2.



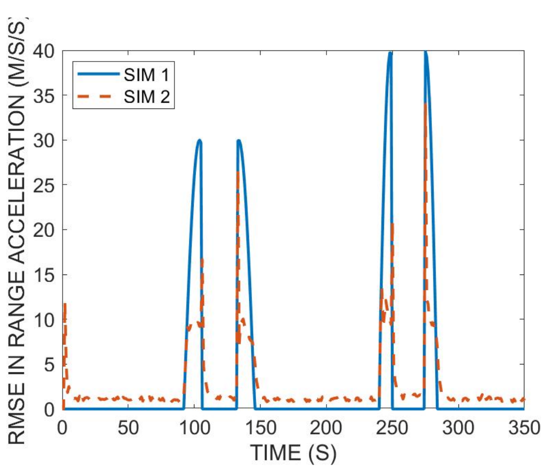


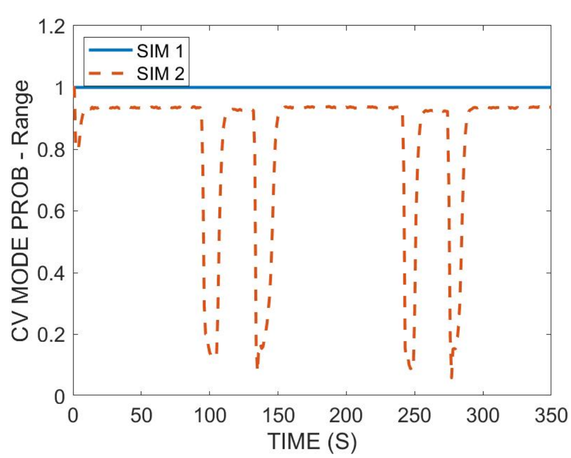
Example 7: NCV Versus IMM CVCA





Example 7: NCV Versus IMM CVCA





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- **Concluding Remarks**

Concluding Remarks

- Track filter is the workhorse of any advanced data association algorithm such as probabilistic data association filter (PDAF), multiple hypothesis tracking (MHT), probabilistic MHT, (PMHT), or particle filter.
- Poorly designed track filter will lead to degraded performance of your data association algorithm and false conclusions regarding relative performances: Poorly tuned track filter will handicap your overall tracking.
- Methods for designing NCV and NCA track filters allow for a desired performance to be achieved.
  - *MinMaxMSE^{pos}*
  - *MaxMSE^{pos}* less than measurement error
- Given the Deterministic Tracking Index Γ_D, maximum acceleration of the target $A_{\max}$, and duration of maneuvers in measurements, upper and lower bounds on the process noise variance σ^2_{ncv} or σ^2_{nca} can be specified.

Concluding Remarks

- NCV Filter Versus NCA Filter
  - If *MinMaxMSE*^{pos} is the sole design criteria, NCV filter is the better option.
  - If maneuvers persist for a sufficient number of measurements to obtain a meaningful estimate of acceleration and improved tracking during a maneuver is desired, the NCA filter may be the better filter for your problem.
  - For most all situations, NCA model should only be considered in an IMM estimator so that the transient error at the end of maneuvers is removed.
- More data does not always lead to better estimates if the filter is poorly designed.
- Effective design methods for algorithms are one of the most important needs in sensor netting.
- Sensor resource allocation: more measurements or better measurements? [11]
  - Estimation Accuracy: better measurements
  - Prediction Accuracy: more measurements
- Use of LFM waveforms significantly improves the mode estimates of an IMM Estimator in three dimensions.
- Additional design methods
  - NCA Radar Tracking with LFM waveforms
  - NCV and NCA radar tracking with FMCW waveforms
  - Multisensor tracking

References

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2. W. D. Blair, "Design of Nearly Constant Velocity Filter for Tracking Maneuvering Targets," *Proceedings of the 2008 International Conference on Information Fusion,* Cologne, Germany, July 2008.
3. Blair, W. D., and Bar-Shalom, Y., "Tracking Maneuvering Targets with Multiple Sensors: Does More Data Always Mean Better Estimates?" *IEEE Transactions Aerospace Electronic Systems*, Vol. 32, No. 1, January 1996, pp. 450-456.
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