DISTRIBUTED SEQUENTIAL LIKELIHOOD RATIO TESTING FOR TRACK EXISTENCE DECISIONS

2024 AESS Virtual DL Webinar

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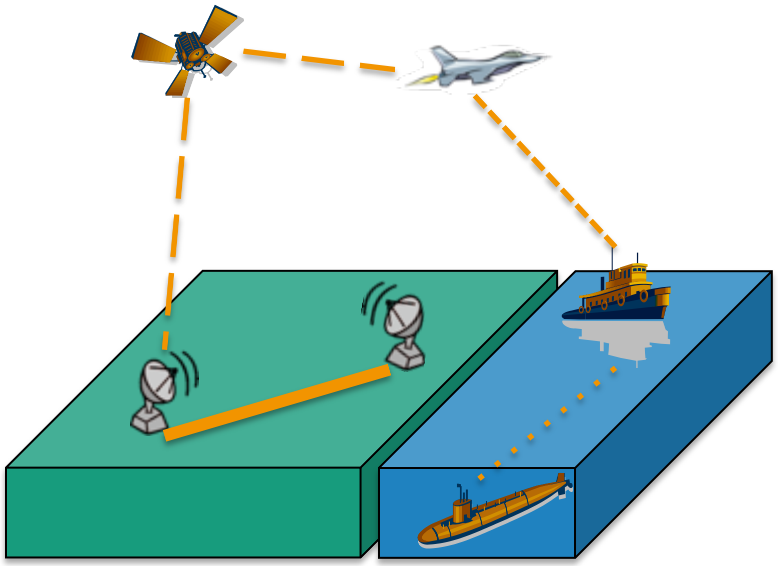
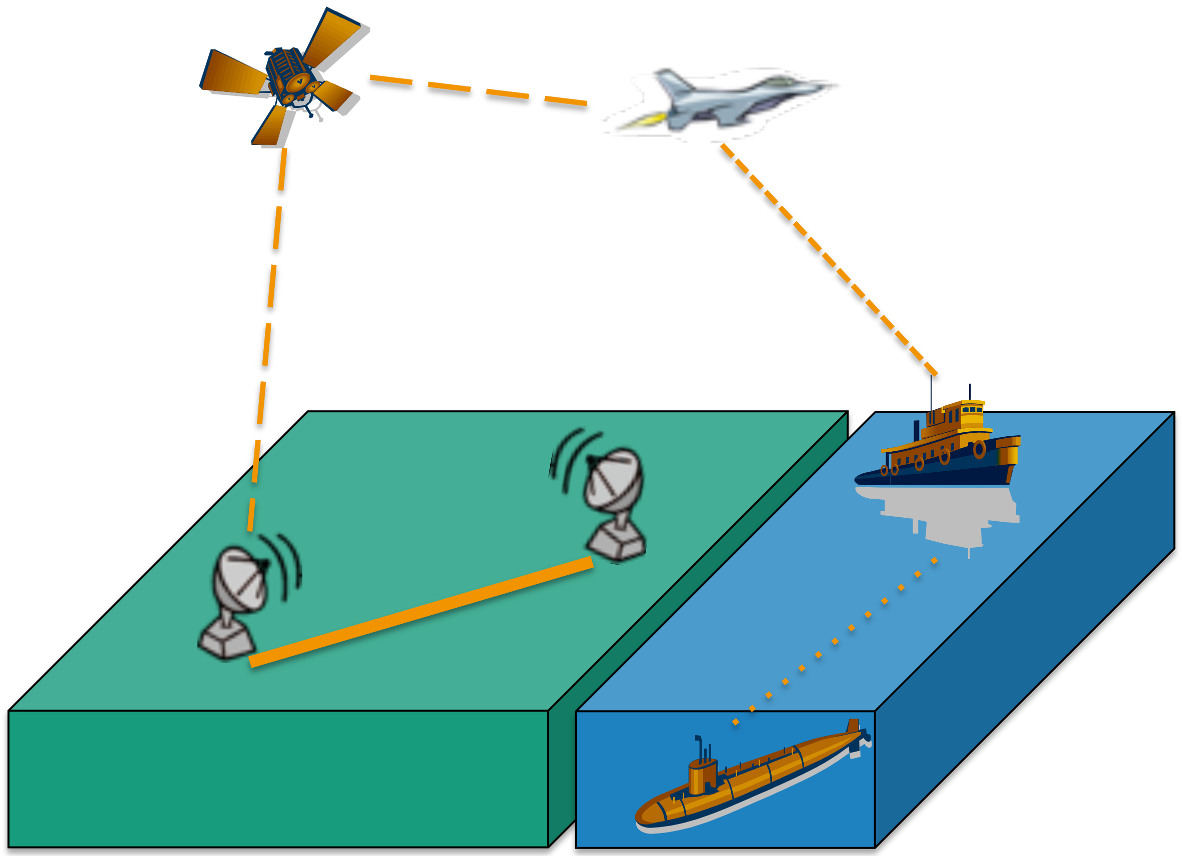


Multi Sensor Fusion with limited Communication

Distributed Kalman filtering has significant advantages in a multi sensor scenario:

- Save bandwidth (preprocessing)
- Distributed calculation
- Full information available at arbitrary instants of time



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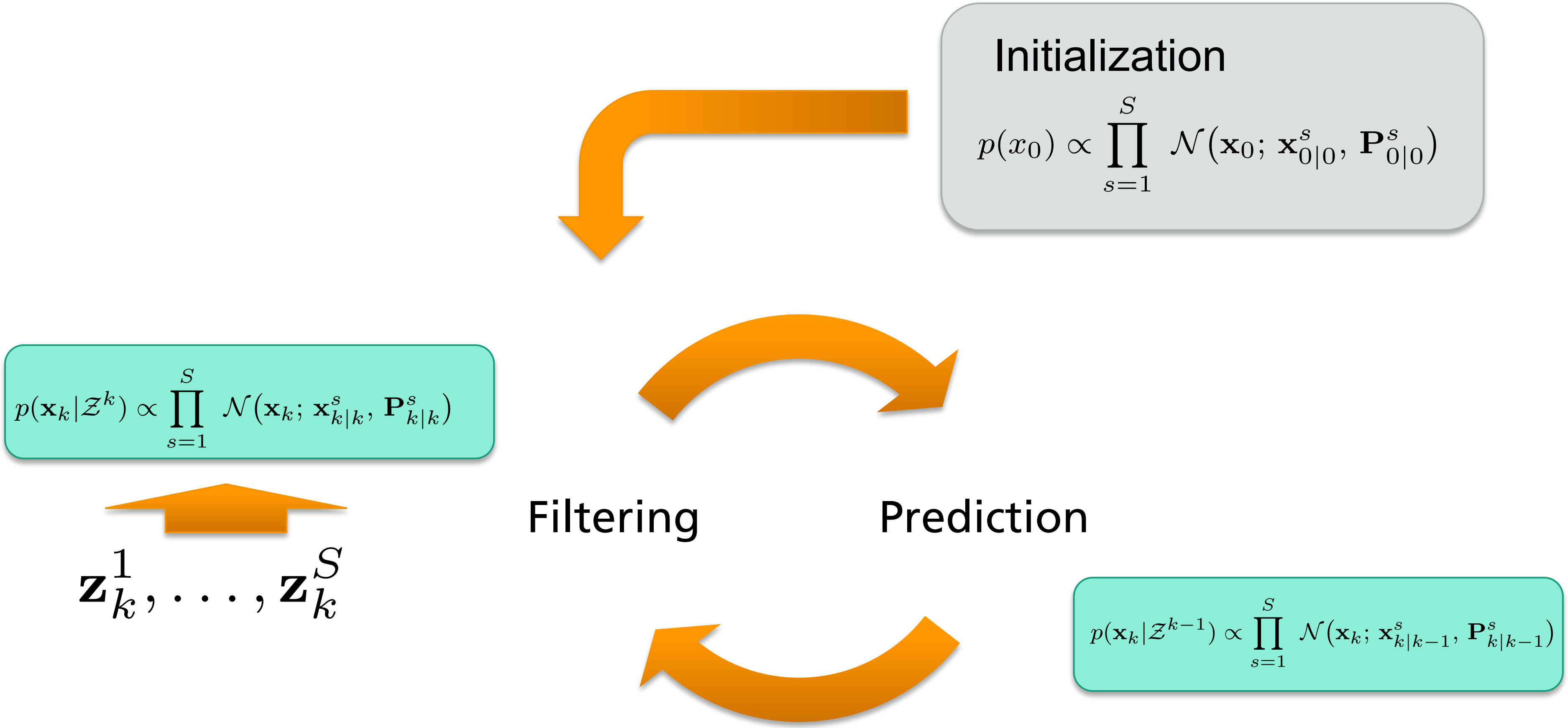
Distributed / Federated / Naïve Kalman Filter
Prediction – Filtering – Cycle



Product Representation

The posterior of S **mutually independent** estimates is given by

Product Representation

$$
\begin{aligned}
p(\mathbf{x}_k | \mathbf{x}^1_{k|k}, \ldots, \mathbf{x}^S_{k|k}) &\propto \prod_{s=1}^{S} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}^s_{k|k},\, \mathbf{P}^s_{k|k}). \\
&= \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k})
\end{aligned}
$$

Naïve Fusion (Convex Combination) is ***exact*** *if and only if cross-covariances are zero.*

$$
\mathbf{x}_{k|k} = \mathbf{P}_{k|k} \left(\sum_{s=1}^{S} (\mathbf{P}^s_{k|k})^{-1} \mathbf{x}^s_{k|k} \right)
$$

$$
\mathbf{P}_{k|k} = \left(\sum_{s=1}^{S} (\mathbf{P}^s_{k|k})^{-1} \right)^{-1}
$$

Product Representation Prediction w/ Relaxed Evolution Model

The prior is calculated by

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \int \mathrm{d}\mathbf{x}_{k-1} p(\mathbf{x}_k, \mathbf{x}_{k-1}|\mathcal{Z}^{k-1})$$
$$= \int \mathrm{d}\mathbf{x}_{k-1} p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathcal{Z}^{k-1}) \cdot p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})$$
$$= \int \mathrm{d}\mathbf{x}_{k-1} p(\mathbf{x}_k|\mathbf{x}_{k-1}) \cdot p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})$$

given in product representation!

$$\propto \prod_{s=1}^{S} \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}^s_{k-1|k-1}\big)$$

transition density

$$= \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{Q}_{k|k-1}\big)$$

$$\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{Q}_{k|k-1}\big) \propto \exp\Big\{-\frac{1}{2}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})^\top \mathbf{Q}^{-1}_{k|k-1}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})\Big\}$$
$$\propto \exp\Big\{-\frac{1}{2}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})^\top S\frac{1}{S}\mathbf{Q}^{-1}_{k|k-1}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})\Big\}$$
$$\propto \exp\Big\{-\frac{1}{2}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})^\top (S\mathbf{Q}_{k|k-1})^{-1}(\mathbf{x}_k - \mathbf{F}_{k|k-1}\mathbf{x}_{k-1})\Big\}^S$$
$$\propto \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, S\mathbf{Q}_{k|k-1}\big)^S$$

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \int \mathrm{d}\mathbf{x}_{k-1} \prod_{s=1}^{S} \Big\{\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, S\mathbf{Q}_{k|k-1}\big)\,\mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}^s_{k-1|k-1}\big)\Big\}$$

Product Formula (1st & 2nd Formulation)

It holds that

$$\mathcal{N}(\mathbf{x};\, \bar{\mathbf{y}},\, \bar{\mathbf{P}}) \cdot \mathcal{N}(\mathbf{z};\, \bar{\mathbf{z}},\, \mathbf{S}) = \mathcal{N}(\mathbf{x};\, \mathbf{y},\, \mathbf{P}) \cdot \mathcal{N}(\mathbf{z};\, \mathbf{H}\mathbf{x},\, \mathbf{R})$$

where

$$\bar{\mathbf{P}} = \begin{cases} \mathbf{P} - \mathbf{W}\mathbf{S}\mathbf{W}^\top \\ (\mathbf{P}^{-1} + \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{H})^{-1} \end{cases}$$

$$\bar{\mathbf{y}} = \begin{cases} \mathbf{y} + \mathbf{W}\boldsymbol{\nu} \\ \bar{\mathbf{P}}\,(\mathbf{P}^{-1}\mathbf{y} + \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{z}) \end{cases}$$

$$\mathbf{S} = \mathbf{H}\mathbf{P}\mathbf{H}^\top + \mathbf{R}$$

$$\mathbf{W} = \mathbf{P}\mathbf{H}^\top \mathbf{S}^{-1}$$

$$\boldsymbol{\nu} = \mathbf{z} - \mathbf{H}\mathbf{y}$$

Apply Product Formula (2nd version)

$$\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, S\mathbf{Q}_{k|k-1}\big) \cdot \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}^s_{k-1|k-1}\big)$$
$$= \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\big) \cdot \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{y}^s,\, \mathbf{Y}^s\big)$$

where

$$\begin{aligned}
\mathbf{x}^s_{k|k-1} &= \mathbf{F}_{k|k-1}\mathbf{x}^s_{k-1|k-1} \\
\mathbf{P}^s_{k|k-1} &= \mathbf{F}_{k|k-1}\mathbf{P}^s_{k-1|k-1}\mathbf{F}^\top_{k|k-1} + S\mathbf{Q}_{k|k-1} \\
\mathbf{y}^s &= \mathbf{Y}^s\big((\mathbf{P}^s_{k|k-1})^{-1}\mathbf{x}^s_{k|k-1} + \mathbf{F}^\top_{k|k-1}(S\mathbf{Q}_{k|k-1})^{-1}\mathbf{x}_k\big) \\
\mathbf{Y}^s &= \big((\mathbf{P}^s_{k|k-1})^{-1} + \mathbf{F}^\top_{k|k-1}(S\mathbf{Q}_{k|k-1})^{-1}\mathbf{F}_{k|k-1}\big)^{-1}
\end{aligned}$$

$$\Rightarrow\quad p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\Big(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\Big) \int \mathrm{d}\mathbf{x}_{k-1} \prod_{s=1}^{S} \big\{\mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{y}^s,\, \mathbf{Y}^s\big)\big\}$$

Federated Kalman Filter Prediction

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right) \int \mathrm{d}\mathbf{x}_{k-1} \prod_{s=1}^{S} \left\{ \mathcal{N}\left(\mathbf{x}_{k-1};\, \mathbf{y}^s,\, \mathbf{Y}^s\right) \right\}$$

$$\approx \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right)$$

The *Federated Kalman Filter* ignores the integral:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right)$$

Federated Kalman Filter Filtering

For the filtering step, we use the fact that measurement noise of the sensors is mutually independent:

$$p(Z_k|\mathbf{x}_k) \propto \prod_{s=1}^{S} p(\mathbf{z}_k^s|\mathbf{x}_k).$$

Using the linear Gaussian assumption, we obtain for the fused posterior:

$$p(\mathbf{x}_k|\mathcal{Z}^k) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{z}_k^s;\, \mathbf{H}_k^s\mathbf{x}_k,\, \mathbf{R}_k^s\right) \mathcal{N}\left(\mathbf{x}_k;\, \tilde{\mathbf{x}}_{k|k-1}^s,\, \tilde{\mathbf{P}}_{k|k-1}\right).$$

And directly obtain

$$p(\mathbf{x}_k|\mathcal{Z}^k) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}_{k|k}^s,\, \mathbf{P}_{k|k}^s\right),$$

$$\begin{aligned}
\mathbf{x}_{k|k}^s &= \tilde{\mathbf{x}}_{k|k-1}^s + \mathbf{W}_{k|k-1}^s \left(\mathbf{z}_k^s - \mathbf{H}_k^s \tilde{\mathbf{x}}_{k|k-1}^s\right) \\
\mathbf{W}_{k|k-1}^s &= \tilde{\mathbf{P}}_{k|k-1} \mathbf{H}_k^{s\,\top} \mathbf{S}_{k|k-1}^{s\,-1} \\
\mathbf{S}_{k|k-1}^s &= \mathbf{H}_k^s \tilde{\mathbf{P}}_{k|k-1} \mathbf{H}_k^{s\,\top} + \mathbf{R}_k^s \\
\mathbf{P}_{k|k}^s &= \tilde{\mathbf{P}}_{k|k-1} - \mathbf{W}_{k|k-1}^s \mathbf{S}_{k|k-1}^s \mathbf{W}_{k|k-1}^{s\,\top}.
\end{aligned}$$

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DISTRIBUTED KALMAN FILTER

Distributed Kalman Filter

Initialization

$$p(x_0) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_0;\, \mathbf{x}^s_{0|0},\, \mathbf{P}^s_{0|0}\right)$$

Prediction

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right)$$

Filtering

$$p(\mathbf{x}_k|\mathcal{Z}^{k}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k},\, \mathbf{P}^s_{k|k}\right)$$

$$\mathbf{z}^1_k, \ldots, \mathbf{z}^S_k$$

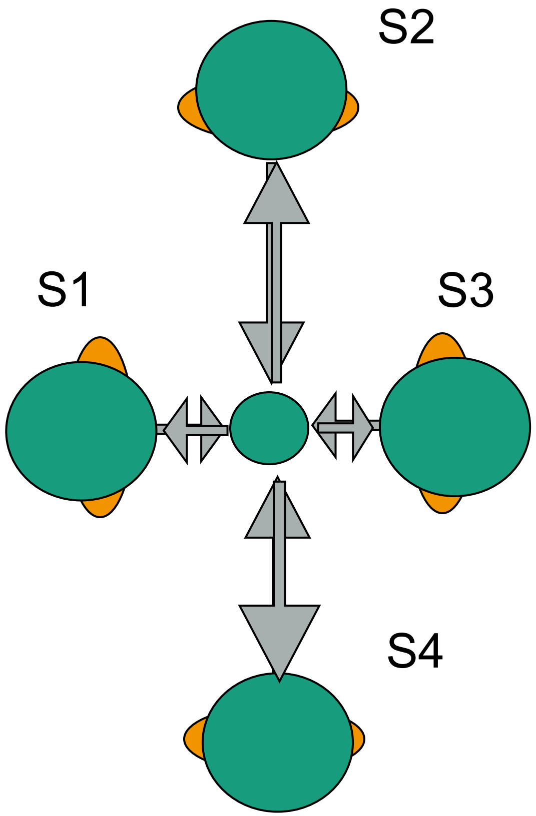
Globalization

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \tilde{\mathbf{P}}_{k|k-1}\right)$$

Globalized Covariance Solution

Exact solution by 'globalizing' the estimate covariance [1]:

$$
\begin{aligned}
p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) &\propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_{k-1};\, \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}^s_{k-1|k-1}\right) \\
&\propto \mathcal{N}\left(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\right) \\
&\propto \mathcal{N}\left(\mathbf{x}_{k-1};\, \mathbf{P}_{k-1|k-1} \sum_{s=1}^{S} (\mathbf{P}^s_{k-1|k-1})^{-1} \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\right) \\
&\propto \mathcal{N}\left(\mathbf{x}_{k-1};\, \frac{1}{S} S\mathbf{P}_{k-1|k-1} \sum_{s=1}^{S} (\mathbf{P}^s_{k-1|k-1})^{-1} \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\right) \\
&\propto \mathcal{N}\left(\mathbf{x}_{k-1};\, \frac{1}{S} \sum_{s=1}^{S} S\mathbf{P}_{k-1|k-1} (\mathbf{P}^s_{k-1|k-1})^{-1} \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\right) \\
&\propto \mathcal{N}\left(\frac{1}{S} \sum_{s=1}^{S} \mathbf{x}_{k-1};\, \frac{1}{S} \sum_{s=1}^{S} S\mathbf{P}_{k-1|k-1} (\mathbf{P}^s_{k-1|k-1})^{-1} \mathbf{x}^s_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\right) \\
&\propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_{k-1};\, S\mathbf{P}_{k-1|k-1} (\mathbf{P}^s_{k-1|k-1})^{-1} \mathbf{x}^s_{k-1|k-1},\, S\mathbf{P}_{k-1|k-1}\right) \\
&\propto: \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_{k-1};\, \tilde{\mathbf{x}}^s_{k-1|k-1},\, \tilde{\mathbf{P}}_{k-1|k-1}\right)
\end{aligned}
$$

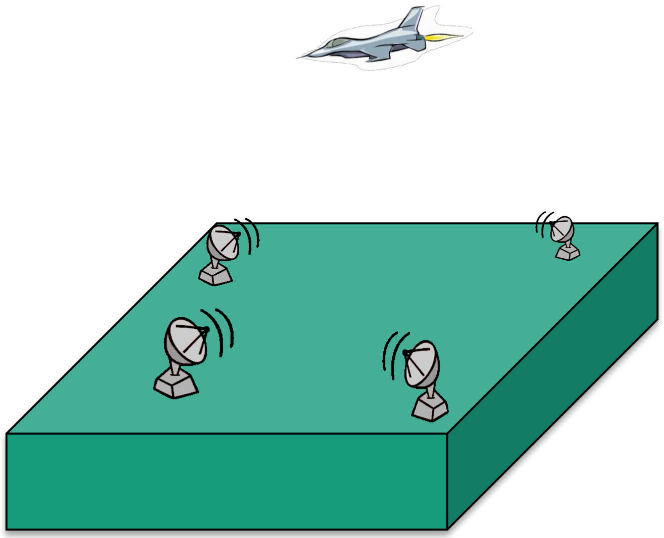


[1] Govaers, F.; Koch, W.; , "Distributed Kalman Filter Fusion at Arbitrary Instants of Time," *Information Fusion (FUSION), 2010 13th Conference on*, 26-29 July 2010

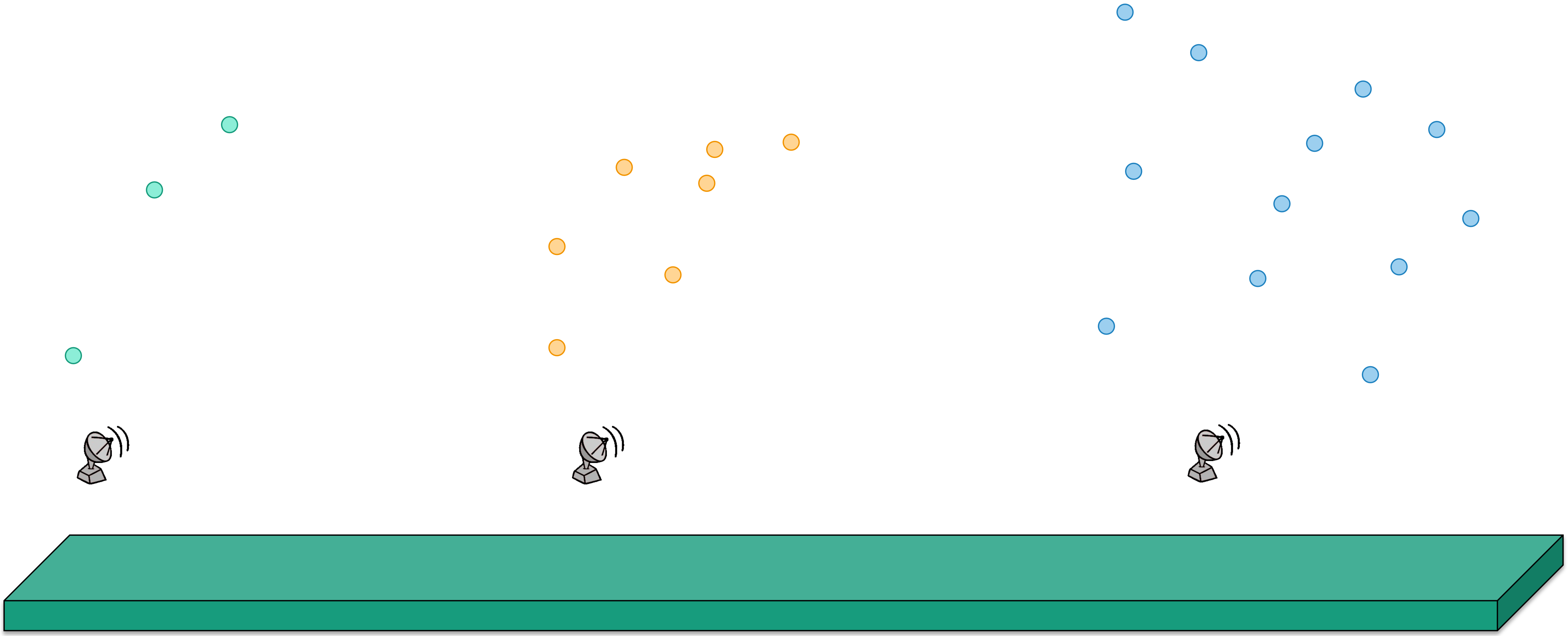
	Distributed Kalman Filter	Federated Kalman Filter	Naïve Fusion
Sensor globalization	$\tilde{\mathbf{x}}^s_{k-1\|k-1} = \tilde{\mathbf{P}}_{k-1\|k-1}(\mathbf{P}^s_{k-1\|k-1})^{-1}\mathbf{x}^s_{k-1\|k-1}$ $\tilde{\mathbf{P}}_{k-1\|k-1} = S\left(\sum_{s=1}^{S}(\mathbf{P}^s_{k-1\|k-1})^{-1}\right)^{-1}$		
Sensor prediction	$\mathbf{x}^s_{k\|k-1} = \mathbf{F}_{k\|k-1}\tilde{\mathbf{x}}^s_{k-1\|k-1}$ $\mathbf{P}^s_{k\|k-1} = \mathbf{F}_{k\|k-1}\tilde{\mathbf{P}}_{k-1\|k-1}\mathbf{F}^\top_{k\|k-1} + S\mathbf{Q}_{k\|k-1}$	$\mathbf{x}^s_{k+1\|k} = \mathbf{F}_{k+1\|k}\mathbf{x}^s_{k\|k}$ $\mathbf{P}^s_{k+1\|k} = \mathbf{F}_{k+1\|k}\mathbf{P}^s_{k\|k}\mathbf{F}^\top_{k+1\|k} + S\mathbf{Q}_{k+1\|k}$	$\mathbf{x}^s_{k\|k-1} = \mathbf{F}_{k\|k-1}\mathbf{x}^s_{k-1\|k-1}$ $\mathbf{P}^s_{k\|k-1} = \mathbf{F}_{k\|k-1}\mathbf{P}^s_{k-1\|k-1}\mathbf{F}^\top_{k\|k-1} + \mathbf{Q}_{k\|k-1}$
Sensor filtering	$\mathbf{x}^s_{k\|k} = \mathbf{x}^s_{k\|k-1} + \mathbf{W}^s_{k\|k-1}\left(\mathbf{z}^s_k - \mathbf{H}^s_k\mathbf{x}^s_{k\|k-1}\right)$ $\mathbf{W}^s_{k\|k-1} = \mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k\mathbf{S}^{s\,-1}_{k\|k-1}$ $\mathbf{S}^s_{k\|k-1} = \mathbf{H}^s_k\mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k + \mathbf{R}^s_k$ $\mathbf{P}^s_{k\|k} = \mathbf{P}^s_{k\|k-1} - \mathbf{W}^s_{k\|k-1}\mathbf{S}^s_{k\|k-1}\mathbf{W}^{s\,\top}_{k\|k-1}.$	$\mathbf{x}^s_{k\|k} = \mathbf{x}^s_{k\|k-1} + \mathbf{W}^s_{k\|k-1}\left(\mathbf{z}^s_k - \mathbf{H}^s_k\mathbf{x}^s_{k\|k-1}\right)$ $\mathbf{W}^s_{k\|k-1} = \mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k\mathbf{S}^{s\,-1}_{k\|k-1}$ $\mathbf{S}^s_{k\|k-1} = \mathbf{H}^s_k\mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k + \mathbf{R}^s_k$ $\mathbf{P}^s_{k\|k} = \mathbf{P}^s_{k\|k-1} - \mathbf{W}^s_{k\|k-1}\mathbf{S}^s_{k\|k-1}\mathbf{W}^{s\,\top}_{k\|k-1}.$	$\mathbf{x}^s_{k\|k} = \mathbf{x}^s_{k\|k-1} + \mathbf{W}^s_{k\|k-1}\left(\mathbf{z}^s_k - \mathbf{H}^s_k\mathbf{x}^s_{k\|k-1}\right)$ $\mathbf{W}^s_{k\|k-1} = \mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k\mathbf{S}^{s\,-1}_{k\|k-1}$ $\mathbf{S}^s_{k\|k-1} = \mathbf{H}^s_k\mathbf{P}^s_{k\|k-1}\mathbf{H}^{s\,\top}_k + \mathbf{R}^s_k$ $\mathbf{P}^s_{k\|k} = \mathbf{P}^s_{k\|k-1} - \mathbf{W}^s_{k\|k-1}\mathbf{S}^s_{k\|k-1}\mathbf{W}^{s\,\top}_{k\|k-1}.$
Fusion center	$\mathbf{x}_{k\|k} = \mathbf{P}_{k\|k}\left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\mathbf{x}^s_{k\|k}\right)$ $\mathbf{P}_{k\|k} = \left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\right)^{-1}$	$\mathbf{x}_{k\|k} = \mathbf{P}_{k\|k}\left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\mathbf{x}^s_{k\|k}\right)$ $\mathbf{P}_{k\|k} = \left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\right)^{-1}$	$\mathbf{x}_{k\|k} = \mathbf{P}_{k\|k}\left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\mathbf{x}^s_{k\|k}\right)$ $\mathbf{P}_{k\|k} = \left(\sum_{s=1}^{S}(\mathbf{P}^s_{k\|k})^{-1}\right)^{-1}$

Target Existence Decision – Distributed

- For automated decision making with respect to track existence, different categories of algorithms exist:
  - Centralized: Send all measurements to the Fusion Center (FC) and decide.
  - Decentralized: Decide locally on each sensor node, send the decisions to all connected neighbors and fuse the received decisions.
  - Distributed: Fuse the local data and compute parameters which are combined in a distinguished fusion center to make a global decision.



Target Existence Decision – Distributed



Likelihood Ratio Test

The Sequential Likelihood Ratio (LR) test is a statistically optimal algorithm to decide between two hypotheses:

- h_1: there is a target.
- h_0: there is no target.

A decision can be made based on two thresholds A and B.

$$\mathrm{LR}(k) = \frac{p(h_1|\mathcal{Z}^k)}{p(h_0|\mathcal{Z}^k)}$$

- $\mathrm{LR}(k) < A$: accept h_0, i.e. delete track
- $\mathrm{LR}(k) > B$: accept h_1, i.e. confirm track
- $A < \mathrm{LR}(k) < B$: continue processing.

Recursive computation of the LR

According to Bayes, one has

$$\mathrm{LR}(k) = \frac{p(\mathcal{Z}^k|h_1)}{p(\mathcal{Z}^k|h_0)}$$

where

$$p(\mathcal{Z}^k|h_i) = p(Z_k|\mathcal{Z}^{k-1}, h_i)\, p(\mathcal{Z}^{k-1}|h_i)$$

$$\overset{i=1}{=} \int \mathrm{d}\mathbf{x}_k\, p(Z_k|\mathbf{x}_k, h_1)\;\; p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)\, p(\mathcal{Z}^{k-1}|h_1)$$

$$\mathrm{LR}(k) = \mathrm{LR}(k-1) \cdot \Lambda(k)$$

$$\Lambda(k) = \frac{\int \mathrm{d}\mathbf{x}_k\, p(Z_k|\mathbf{x}_k, h_1)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)}{p(Z_k|h_0)}$$

Distributed Kalman Filter (DKF)

The Distributed Kalman Filter (DKF) achieves a product representation of local estimate parameters also, if process noise is present:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\big)$$

- The DKF computes the exact product representation but cannot be applied when the local covariances are data dependent.
- The FKF and Naïve Fusion yield approximately a product representation:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \approx \frac{1}{c_{k|k-1}} \prod_{s=1}^{S} \mathcal{N}\Big(\mathbf{x}_k; \mathbf{x}^s_{k|k-1}, \mathbf{P}^s_{k|k-1}\Big)$$

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Distributed Sequential Likelihood Ratio Test

The LR score at time k is given by

$$\mathrm{LR}(k) = \mathrm{LR}(k-1) \cdot \Lambda(k)$$

$$\Lambda(k) = \frac{\int \mathrm{d}\mathbf{x}_k \, p(Z_k|\mathbf{x}_k, h_1) \, p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)}{p(Z_k|h_0)}$$

where

$$p(Z_k|h_0) = |\mathrm{FoV}|^{-m} p_F(m)$$

$$p(Z_k|\mathbf{x}_k, h_1) = (|\mathrm{FoV}|^{-m} p_F(m))\left((1-p_D) + \frac{p_D}{\rho_F} \sum_{j=1}^{m} \mathcal{N}\big(\mathbf{z}_j;\, \mathbf{H}_k \mathbf{x}_k,\, \mathbf{R}_k\big)\right)$$

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \frac{1}{c_{k|k-1}} \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right)$$

Now, the normalization constant is important!

DKF Normalization Constant

The normalization constant is given by

$$c_{k|k-1} = \int \mathrm{d}\mathbf{x}_k \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right).$$

Algebraic manipulations yield

$$c_{k|k-1} = \prod_{s=1}^{S-1} \mathcal{N}\left(\mathbf{x}^{s+1}_{k|k-1};\, \mathbf{x}^{(1:s)}_{k|k-1},\, \mathbf{P}^{(1:s)}_{k|k-1} + \mathbf{P}^{s}_{k|k-1}\right)$$

$$\mathbf{x}^{(1:s)}_{k|k-1} = \mathbf{P}^{(1:s)}_{k|k-1} \sum_{i=1}^{s} (\mathbf{P}^{i}_{k|k-1})^{-1} \mathbf{x}^{i}_{k|k-1},$$

$$\mathbf{P}^{(1:s)}_{k|k-1} = \left(\sum_{i=1}^{s} (\mathbf{P}^{i}_{k|k-1})^{-1} \right)^{-1}.$$

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Sequential LR Update for DKF

The updating factor of the LR is given by

$$\Lambda(k) = \frac{1}{c_{k|k-1}} \int \mathrm{d}\mathbf{x}_k \prod_{s=1}^{S} \Big\{ \Big((1-p_D) + \frac{p_D}{\rho_F} \sum_{j=1}^{m_s} \mathcal{N}\big(\mathbf{z}_k^{j,s};\, \mathbf{H}_k^s \mathbf{x}_k,\, \mathbf{R}_k^s\big) \Big) \cdot \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1}^s,\, \mathbf{P}_{k|k-1}^s\big) \Big\}$$

An application of the product formula yields

unnormalized hypotheses weights

$$\Lambda(k) = \frac{1}{c_{k|k-1}} \int \mathrm{d}\mathbf{x}_k \Big\{ \prod_{s=1}^{S} \sum_{j=0}^{m_s} p^{\star\, j,s} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k}^{j,s},\, \mathbf{P}_{k|k}^{j,s}\big) \Big\}$$

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Normalization and Moment Matching

The normalized weights are given by

$$p^{j,s} = \frac{p^{\star\, j,s}}{\bar{p}^s} \qquad \bar{p}^s = \sum_{j=0}^{m_s} p^{\star\, j,s}.$$

Therefore:

$$\sum_{j=0}^{m_s} p^{\star\, j,s} \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{j,s}, \mathbf{P}_{k|k}^{j,s}\right) = \bar{p}^s \underbrace{\sum_{j=0}^{m_s} p^{j,s} \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{j,s}, \mathbf{P}_{k|k}^{j,s}\right)}_{\overset{\text{MM}}{\approx} \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{s}, \mathbf{P}_{k|k}^{s}\right)}$$

Computation of Lambda

As a result one obtains

$$\Lambda(k) = \frac{c_{k|k}}{c_{k|k-1}} \prod_{s=1}^{S} \bar{p}^s \int \mathrm{d}\mathbf{x}_k \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$$

$$= \frac{c_{k|k}}{c_{k|k-1}} \prod_{s=1}^{S} \bar{p}^s$$

where the posterior normalization constant is given by

$$c_{k|k} = \prod_{s=1}^{S-1} \mathcal{N}\big(\mathbf{x}_{k|k}^{s+1};\, \mathbf{x}_{k|k}^{(1:s)},\, \mathbf{P}_{k|k}^{(1:s)} + \mathbf{P}_{k|k}^{s}\big)$$

Conclusion: Distributed Track Existence Decision

Local Sensor Nodes

Prediction: Relaxed Evolution Model

$$\mathbf{x}^s_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}^s_{k|k-1},$$
$$\mathbf{P}^s_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}^s_{k|k-1}\mathbf{F}^\top_{k|k-1} + S\mathbf{Q}_{k|k-1}$$

Filtering:

- Update state parameters with EKF / MHT / PDAD / …
- calculate decision contribution

$$p^{\star j,s} = \begin{cases}(1-p_D) \\ \frac{p_D}{\rho_F}\mathcal{N}\left(\mathbf{z}^{j,s}_k; \mathbf{H}^s_k\mathbf{x}^s_{k|k-1}, \mathbf{S}^s_k\right)\end{cases}$$

$$\bar{p}^s = \sum_{j=0}^{m_s} p^{\star j,s}.$$

Tx:

$$\mathbf{x}^s_{k|k}\cdot \quad \mathbf{P}^s_{k|k} \quad \bar{p}^s$$

Fusion Center

Prediction: calculate prior constants using the Relaxed Evolution Model and the previous transmission:

$$c_{k|k-1} = \prod_{s=1}^{S-1}\mathcal{N}\left(\mathbf{x}^{s+1}_{k|k-1}; \mathbf{x}^{(1:s)}_{k|k-1}, \mathbf{P}^{(1:s)}_{k|k-1} + \mathbf{P}^s_{k|k-1}\right),$$

Filtering: calculate posterior constants using the new transmissions. Update LR score:

$$\mathrm{LR}(k) = \Lambda(k)\cdot\mathrm{LR}(k-1)$$

$$\Lambda(k) = \frac{c_{k|k}}{c_{k|k-1}}\prod_{s=1}^{S}\bar{p}^s$$

NUMERICAL EXAMPLES

Simulation Setup

For the evaluation, a realistic multi-radar scenario has been chosen:

- 4 radars arranged along a circle of about 13 km
- Poisson distributed FA with mean 5 per sensor per scan
- The target, if present, has a process noise of psd = 10
- Probability of detection is $p_D = 0.2, 0.5$ and $p_D = 0.9$.
- A no target scenario is also considered
- We compare against:
  - Centralized processing (CKF) for LR calculation (optimal)
  - Decentralized mean of all local LR scores (LKF)

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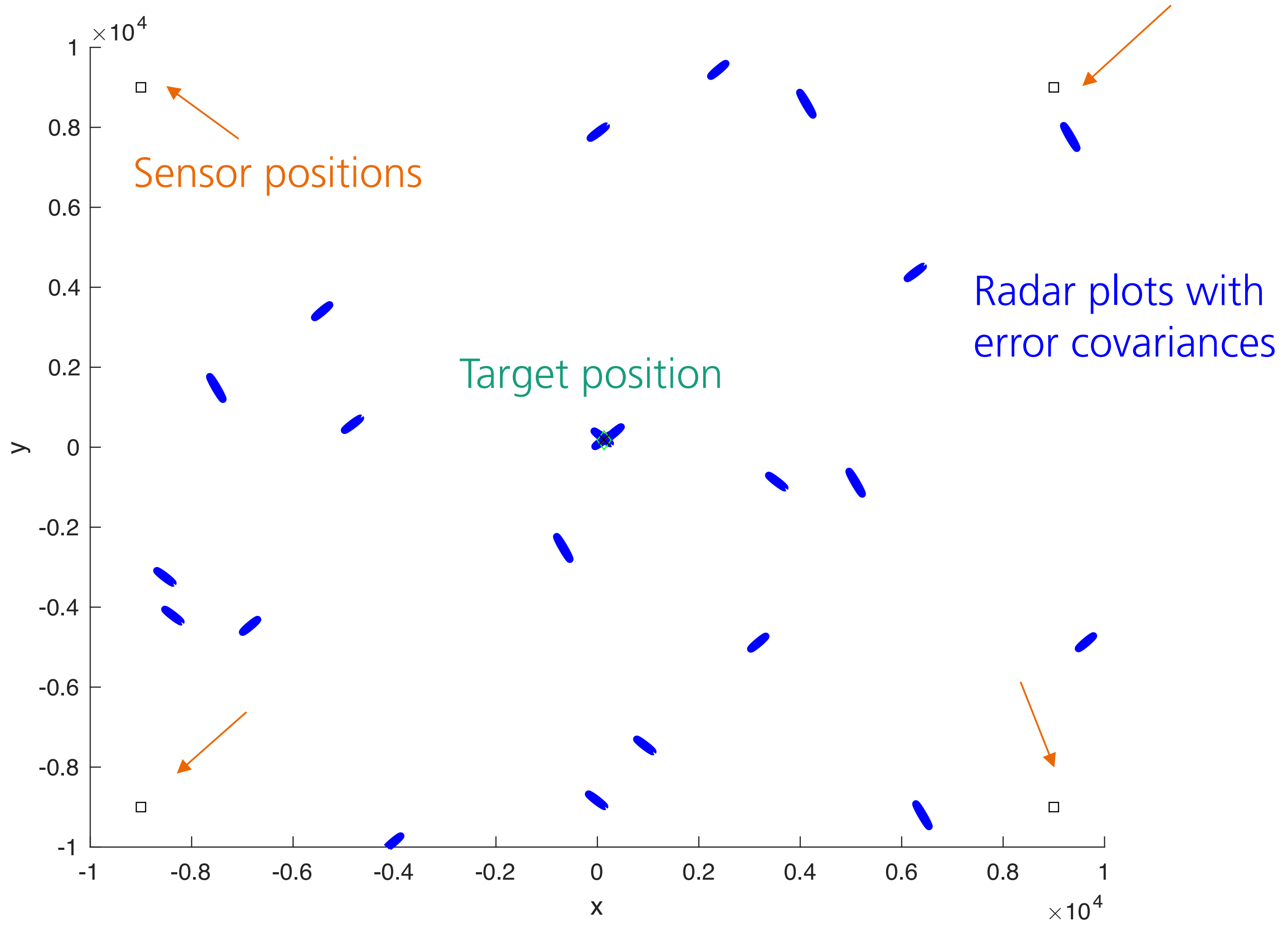
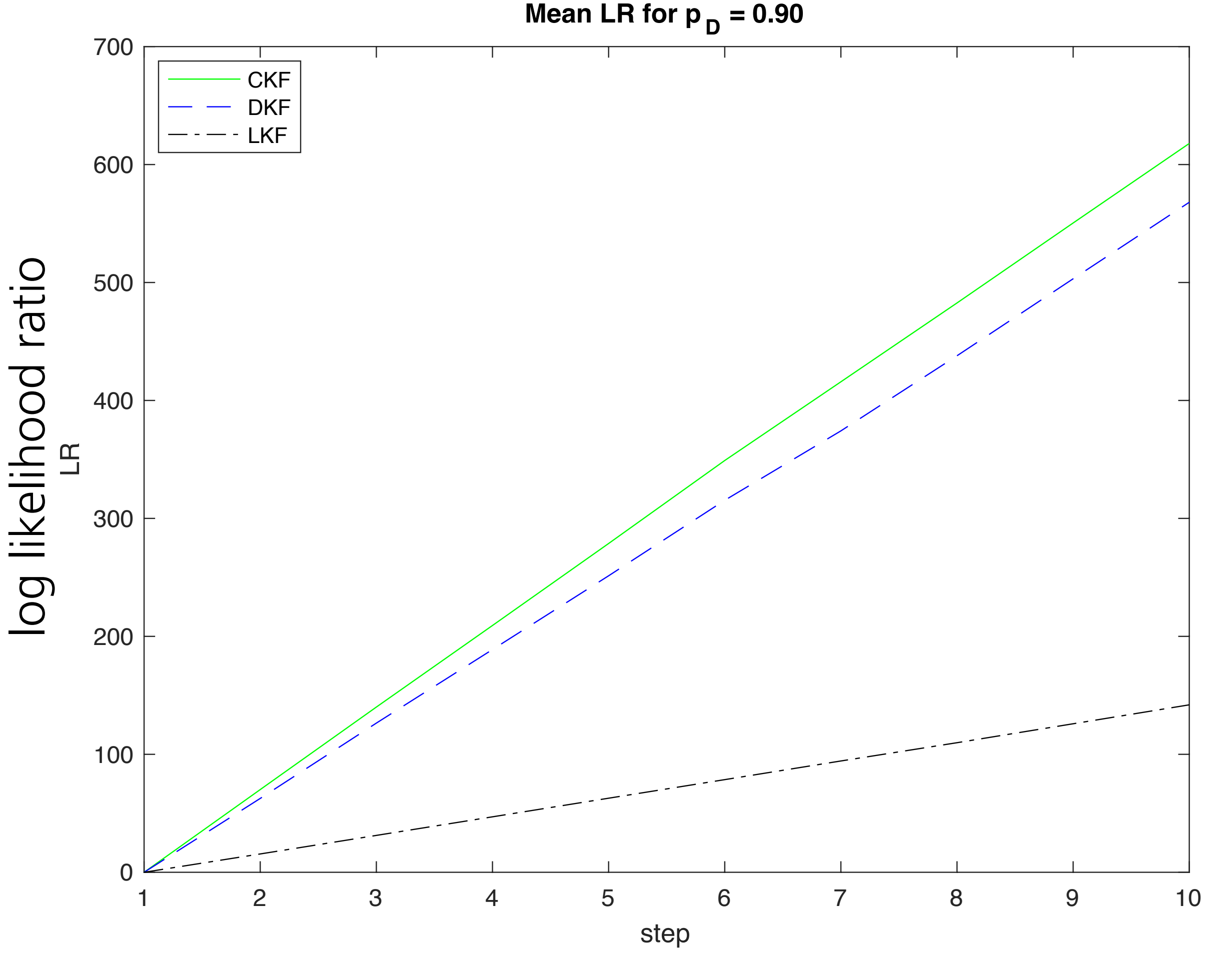
Plot of a single scan



Fraunhofer FKIE

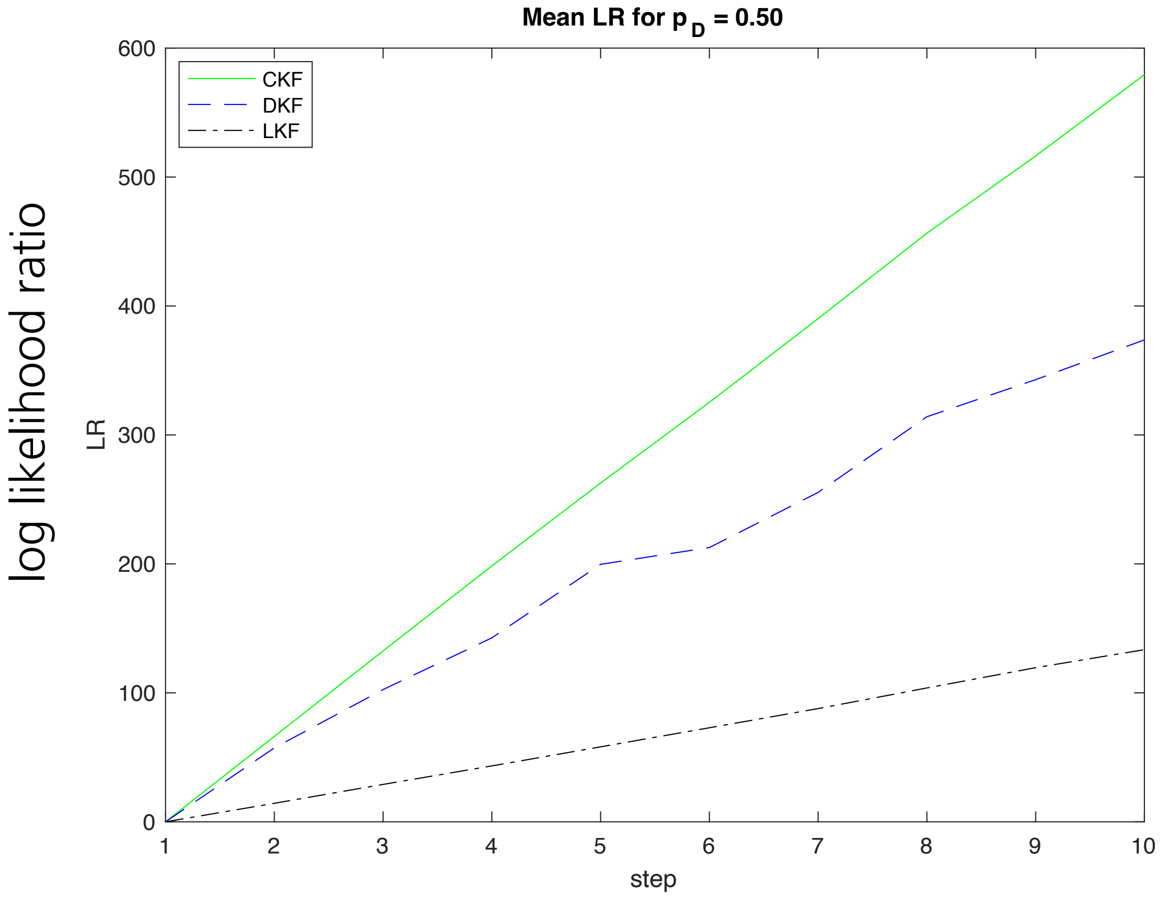
Numerical Results of the LR Scores

$p_D = 0.9$



Numerical Results of the LR Scores

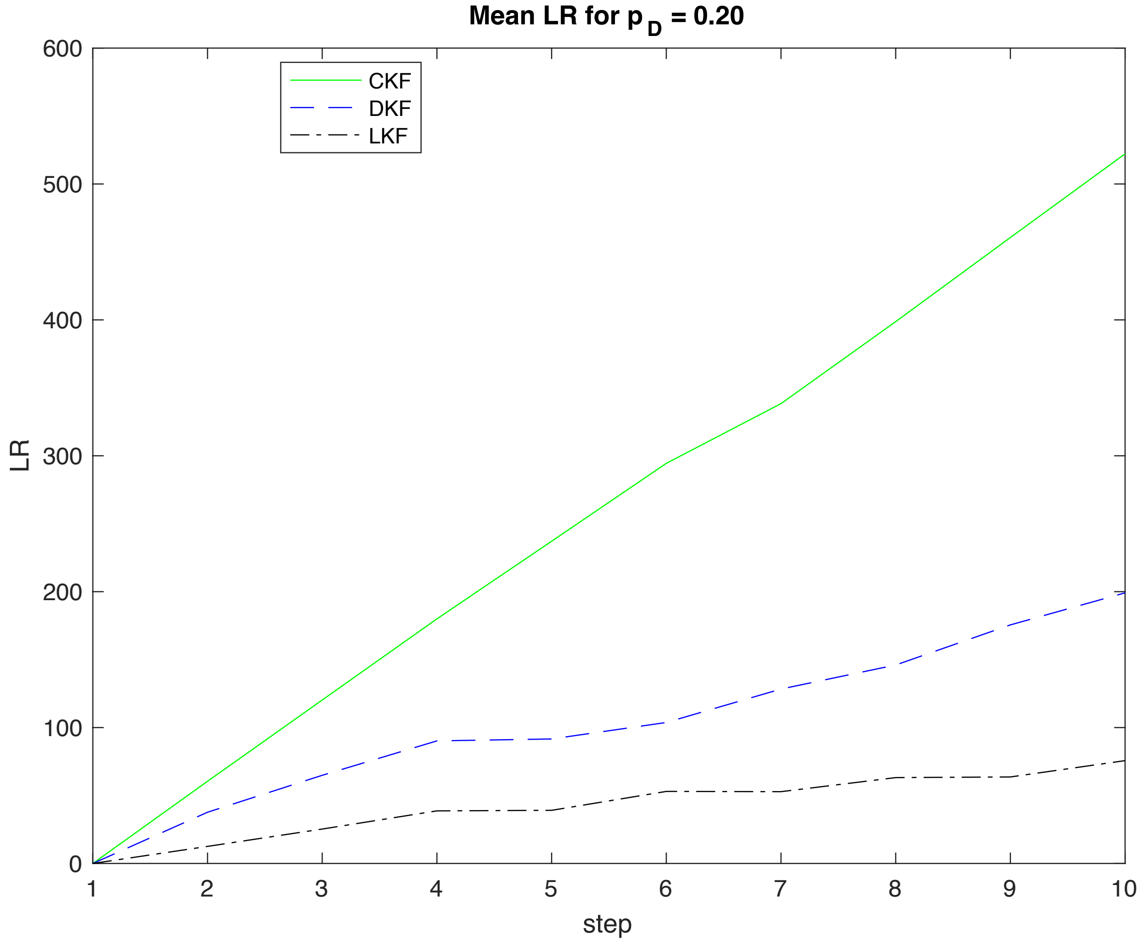
$$p_D = 0.5$$



Numerical Results of the LR Scores

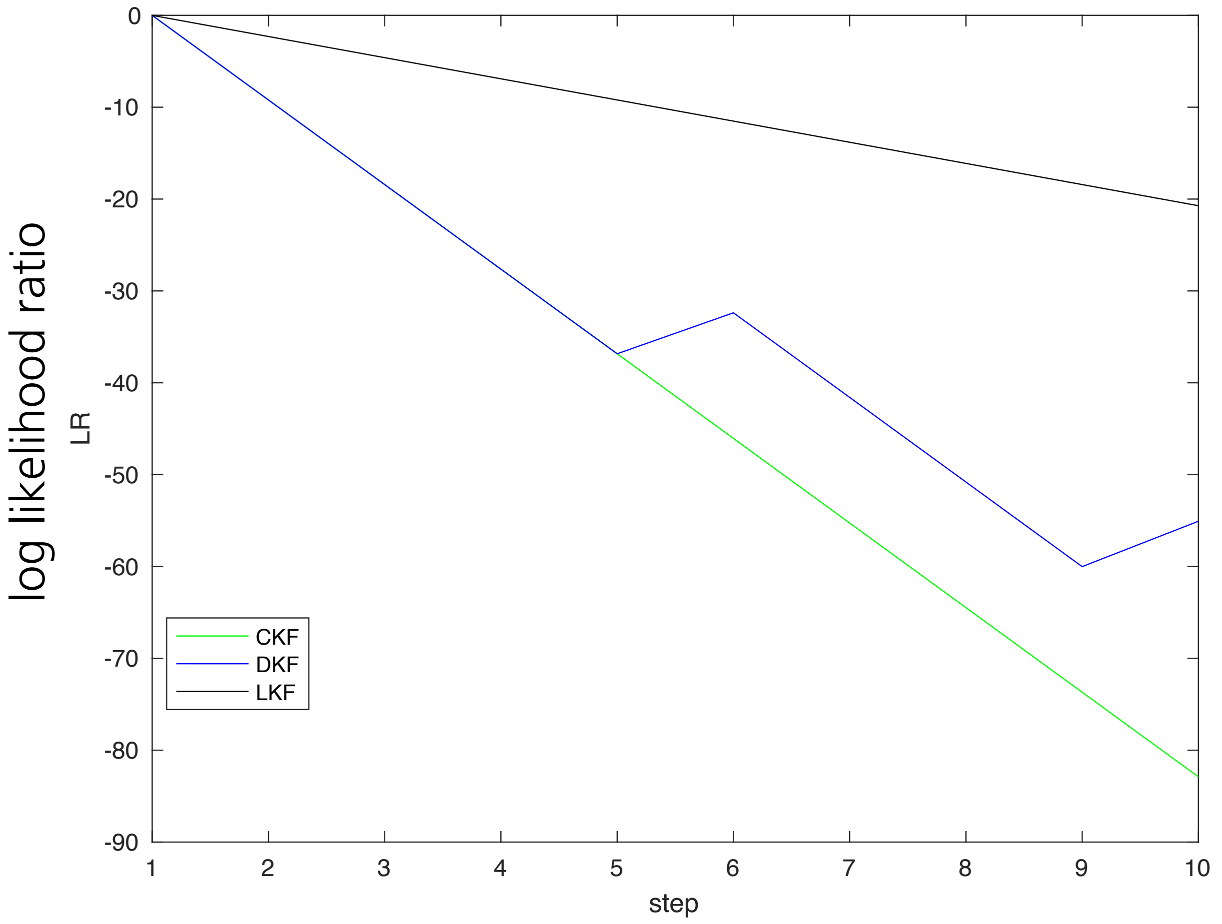
$$p_D = 0.2$$

log likelihood ratio



Numerical Results of the LR Scores

No target



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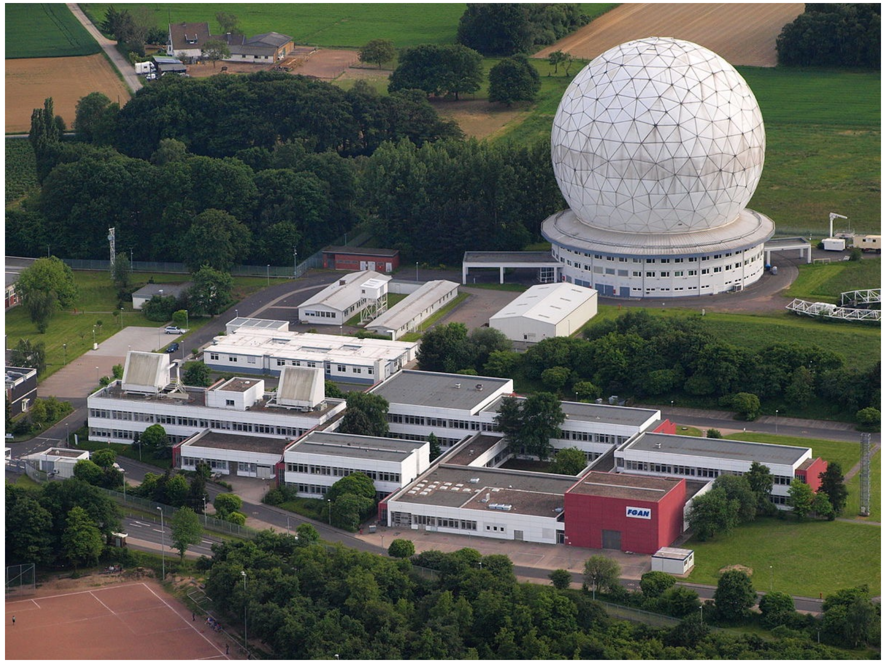
Conclusion

- Distributed Sequential Likelihood Ratio for decision on target detection has been presented.
- Fusion center computes LR score based on single real valued parameter from each sensor.
- The distributed calculation clearly performs better than averaging the local LR scores even with identical sensors parameters.
- The method can well be applied to real world applications.



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